

From Input-Output Matrixes to Agent-Based Models: A Case Study on Carbon Credits in a Local Economy

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Abstract: Analytical macroeconomic scenarios are currently the most common approach to assist in the development and evaluation of economic policies. Reproducing and evaluating the results found by analytic models is one major hurdle to be overcome by social simulation on its early development in any specific knowledge area. This work describes an initial step toward moving from algebraic input-output economic models to agent-based models, in order to get more flexibility, adding decision capabilities to the agents and exploring more complex scenarios. We study economic scenarios of carbon credits for reducing deforestation in a region of Para state, in the Brazilian Amazonia. The objective is to investigate the underlying assumptions of the analytic model through an agent-based approach, deriving new challenges to be tackled by future agent-based models.

1 Introduction

The main challenge of social simulation on its early development in any specific knowledge area is firstly on how to reproduce and evaluate the results found by analytic or even descriptive models. We can find examples of this kind of study in areas such as residential land markets [11, 8], common pool resources [3], and predator-prey dynamics [26]. Bringing these areas to the computational world, forcing the models to be consistent, complete, and unambiguous [10], allows us to formalise assumptions and make testable hypothesis.

This work also concerns an already developed literature, focusing on macroeconomics. The target is supply chain scenarios for studying economic development, which has been having many studies on the last sixty years. Mathematically, supply chains are bi-directed graphs representing the economic sectors involved in a given economy and their relations. One can describe a local, regional, national or even global economy from the first input producers until the final consumers or the export sector, passing through every economic sector that adds more value to the products, according to the objective of the study. Leontief build the basis for the so-called input-output matrix [15],

which has been used as benchmarking of Gross Domestic Products (GDP) of countries. Using input-output matrixes, it also possible to analyse future scenarios for the economic development of the interested study region.

On the other side, most agent-based models that work with economic chains focus mainly on their *management*. The basic framework over which the great majority of agent-based models of economic chains are built is the well-known Beer Distribution Game [18]. Related works in the literature are mostly interested in making the chain working properly, studying learning [14, 5, 27, 22, 24, 9] trust [21, 13], information sharing [20], or comparing agent-based modelling with other paradigms [23, 16]. This happens because transportation time is significant, and positive as well as negative stocks imply in costs. Because of that, most of these works end up not concerning real world economic data.

In this work, we suppose the economic chain already works properly: there is no need to synchronise stocks because the time scale is larger: instead of working with daily productions, we study the economic development in a yearly scale. Within this context, we can focus on the study of scenarios for a given economy. Specifically, we study scenarios of carbon credits for reducing deforestation in a region of Para state, in Brazilian Amazonia. We rely upon input-output matrixes that can describe whole economies, and propose a strategy for creating agent-based models from input-output matrixes. This work describes an initial step toward moving from algebraic input-output economic models to agent-based models, in order to get more flexibility, adding decision capabilities to the agents and exploring more complex scenarios. We investigate the underlying assumptions of the analytic model through an agent-based approach.

This paper is written as follows. In Section 2 we present the main ideas related to input-output matrixes, and on how they can be used to study future economic scenarios. The proposed agent-based model is described in Section 3. A study case for testing the proposed model is presented in Section 4, and the results are shown in Section 5. Finally, in Section 6 follow the conclusions and future directions of this work.

2 Input-output matrixes

An input-output matrix describes the sectors of a given economy, and their economic exchanges. It does not represent a screenshot of a given moment, but it is a summary of an economy in a time interval. Usually, the model uses a yearly time scale, because it is common to have cyclic

events in lower temporal scales. Different economic sectors can be grouped in one single actor, according to the objectives of the study, as long as they keep a minimum internal homogeneity on their economic behaviour, or when there is not any data available to split them.

Table **Erro! Fonte de referência não encontrada.** shows an example of an input-output matrix. This matrix is a result of a long effort, involving data from more than a dozen sources. It describes the economic sectors of Southeast Para State, in the Brazilian Amazonia, as well as their economic relations. Lines of the matrix represent sellers, columns represent buyers, and the values of each position are exchanges between them. Note that there are more buyers than sellers because there exist a final demand to the economy. This demand can be local, generated by the necessities of the families and of capital formation, or it can be from other geographic regions, such as the state or the country. This input-output matrix also contains economic indicators for each sector in the lowest part, which are *profit*, *taxes*, *employment*, and *carbon equivalent emissions*. In this specific case, carbon emissions are calculated only to alpha producers, which represent the primary sectors of the economy (small and large farmers and mining).

Based on this matrix, it is possible to estimate the impacts on an economy by calculating the Inverse of Leontief matrix, represented by $(I-A)^{-1}$, where I is the identity matrix and A is a normalised input-output matrix. The inverse matrix points out the sum of the *direct*, *indirect*, and *induced* effects on the economy. If there is an increase in the final demand for a particular product, we can assume that it will generate a *direct* increase in the output of that product because producers react to meet the increased demand. As producers increase their output, it will cause an *indirect* increase in demand on their suppliers and so on down the supply chain. As result of the direct and indirect effects, the level of household income throughout the economy will also increase because of the increased employment. A proportion of this increased income will be spent again on final goods and services: this is the *induced* effect. The ability to quantify these multiplier effects is important as it allows economic impact analyses to be carried out on an economy.

From the inverse of Leontief matrix, economic scenarios can be obtained by multiplying it by a column matrix representing the expected demands. These scenarios are usually future plans of investment from public policies or even from great companies. The resulting column matrix contains the effects of the demands over each economic sector.

There is a couple of ways to calculate the inverse of Leontief. Initially, it used to be calculated by traditional methods for matrix inversion but computationally it is time demanding. Due to its particular properties (inversion of a normalised matrix), it is possible to

have simpler but interesting solution to calculate the inversion. Waugh describes a simple method to calculate an approximation of $(I-A)^{-1}$ [25]. The algorithm is based on the sum of the following power series:

$$I+A+A^2+A^3+A^4+\dots+A^{m-1}+\dots$$

The algorithm uses I as initial demand, and then calculates the indirect demand by summing A for each time the money circulates in the economy, each time circulating less than in the previous time step. The algorithm is approximated because we cannot sum the whole power series. In the next section we present an agent-based model that has similarities with the work of Waugh. After that, we can study its assumptions and limitations, and propose further development to work with agent-based models to study economic scenarios.

3 The proposed model

The objective of the model presented on this paper is to reproduce the same general behaviour of the traditional input-output economic model by using an agent-based approach. The idea is to make money circulate in the economy; there is no circulation of production. The model takes an input-output matrix as parameter, creates a society of agents, and connects them according to the economic relations described within the input-output matrix.

Observing the input-output matrix in Table **Erro! Fonte de referência não encontrada.**, we can see directly that the agents can be extracted from its columns. As the lines only replicate a subset of the columns, agents that do not come up in the lines are considered exogenous or partially exogenous. Note only that, although some column names are in plural form, such as *farmers* and *families*, each of them is represented by a single agent. Finally, one last agent is necessary to complete the society: *the government*. It is responsible for receiving taxes and may use this money with public policies and other expenses.

Once we have agents extracted from the input-output matrix, we need to connect them to represent the economic relations. Non-zero values in the matrix indicate that there is a connection between agents because the column-agent requires new inputs from the line-agent. Every agent is then connected to the ones in the respective lines of the non-zero values, and also to the government and the families. Note only that one agent may or not be connected to itself. Summing up, we have a directed graph, where nodes represent agents and connections represent economic relations from one agent to another.

However, the connections by themselves are not enough to describe the economic relations between agents. We need to define *weights*, representing the proportion of inputs one agent spend with others in order to produce its output. The weights of the connections are thus proportional to the expenses of that agent in the input-output table, normalised to one. Clearly, the sum to be normalised excludes employment and CO₂ balance because they are not interchangeable values, but results from the underlying economic process. Therefore, both are calculated as proportions to the overall input one agent takes.

Let us present an example on how the behaviour of an agent is generated according to a column of a synthetic input-output matrix, shown in Table 1. The agent representing sector *a* is connected to itself, two other sectors (*b* and *c*), the government (because of the taxes), and the families (because of the salaries). The weights of these connections are shown in the right column, summing up one. Finally, the jobs and CO₂ emissions are normalised by using Total.

Table 1: A column of a synthetic input-output matrix and its normalisation.

	a	Normalised
a	400	0.08
b	1,000	0.20
c	600	0.12
Taxes	500	0.10
Profit	1,000	0.20
Salaries	1,500	0.30
Total	5,000	1.00
Jobs	50	0.01
CO₂ emissions (T)	100	0.02

Given that, if agent *a* receives a payment of, for instance, \$1,000 for its inputs, it will execute the following actions:

1. Separate the profit of \$200;
2. Pay \$100 as taxes to the government;
3. Pay \$300 as salaries to the families;
4. Create ten jobs;
5. Emit 2T of CO₂; and

6. Send \$80 to itself, \$200 to *b*, and \$120 to *c*.

The behaviour of the agent above is simply reactive. It waits for some external demand from another agent and then generate its production, creating a new demand and activating its neighbours, always proportional to the values of the original input-output matrix.

All the agents of the model behave in the same way of the previous description. It means that this model is completely top-down, with the initial demand coming from exogenous agents. With this, the model requires an external demand generated automatically at the beginning of the simulation. After this, the model executes until the internal demand becomes very low. Consequently, we need to have other agents to produce the first and exogenous demand of the model. In the case of Table **Erro! Fonte de referência não encontrada.**, *capital formation*, *state*, and *country* simply generate an initial demand, while *families* also generates more demand coming from the payment of salaries. Therefore, everything that goes to the families but salaries is considered as government expenses, such as public workers, retirements, and social programs, which are modeled as exogenous factors.

4 A study case: carbon balance in a critical region of Amazonia

The study case of this work is based on the experiments of Costa [6], who created the input-output matrix shown in Table **Erro! Fonte de referência não encontrada.** His work is centred on a critical region of Brazilian Amazonia, the southeast of Pará state. In 2004, this region was responsible for 33% of Pará's GDP and of 12% of Brazilian's North Region. There are medium and large farms, with an estimative of a highly carbon dependent agricultural production area growing from 10.2 to 14 milion Ha between 1995 and 2004 [7]. Concerning mineral economy, Vale is the main company acting within the region, exploring ferrous metals and having strong investments in the region since the 80's. In 2009, Pará state was responsible for more than half deforestation of the Brazilian Amazonia, with around 2.5 times more than the second state [12]. Understanding the dynamics that occur in this region is important because it is highly representative in terms of the economic and social processes that are taking place in the whole Amazonia.

Costa's work critics the Stern review [19], in the sense that it has a very aggregated point of view, and does not study the economic multipliers and their impacts on the economy. Stern by himself admits

the limitations of the approach, saying that “[t]he opportunity costs to national GDP would be somewhat higher, as these would include value added activities in country and export tariffs.” The objective then is to analyse the real impacts of carbon credits in this local economy by analysing a couple public policies for reducing CO₂ emissions.

Costa proposes four economic scenarios to study the effects of carbon credits on salary, profit, employment and carbon emissions. Table 2 summarises the exogenous inputs of the scenarios, which are:

- S₁**: Rural producers are compensated with 50% of their overall anual profits (R\$435.14 milions), which are going to be spent in retails and services. The credits are given in exchange for cutting off 50% of the agricultural production. This scenario supposes that agents will not move away or arrive in the region.
- S₂**: Rural producers reduce their agricultural production in 50%, being compensated with 50% of their anual profits. However, new producers arrive the region and replace the reduced production.
- S₃**: There is a conversion of the productive model from those who emit more carbon to those that emit less carbon. Resources of R\$435.14 milions are spent with knowledge transfer and subsidies in order to enforce the conversion.
- S₄**: The same as S₁, but with the non-agricultural primary production growing strongly, with around R\$6,5 bilions of new investments in this area.

Table 2: Scenarios proposed by [6], in R\$ milions of 2004.

	S ₁	S ₂	S ₃	S ₄
Large farmers	-367.67	-	-735.33	-367.67
Small farmers	-287.21	-	735.33	-287.21
Mining	-	-	-	6,563.05
Retail/Services	435.14	435.14	435.14	435.14

5 Experiments

The model was implemented using TerraME [4]. This section presents the experiments and discusses the results.

In the first experiment, we do not study the scenarios proposed by Costa yet, but whether the model can work properly. The objective is to compare the simulation results with the data of the input-output matrix presented in Table **Erro! Fonte de referência não encontrada.** We use the external demand as input for the model and study whether the model is capable to reproduce the economic indicators of the original input-output table. To do so, we use the four columns of final demand as input, only removing the total amount of salaries from the demand of the families, because this value will endogenously circulate in the economy. Initially, the model generates the exogenous demand and circulates the money among the different economic sectors until no agent has more than R\$0.01 to spend.

The input-output matrix totals and the respective results of the simulations are shown in Table 3. The empirical values correspond to the sum of the respective lines of the input-output table. We can see that the achieved results are very close to the input-output table, with maximum error of 0.69, in the total demand. Therefore, the model can reproduce the overall demand and generate the same economic indicators using only the initial exogenous demand.

Table 3: Results replicating the same demand of the original input-output matrix.

	Input-output Matrix	Simulation
Salaries	1,811.70	1,811.45
Profit	7,921.40	7,921.45
Taxes	1,275.20	1,275.18
Employment	368.20	368.19
Carbon emissions	293.20	293.19
Total demand	25,752.10	25,751.41

In the second experiment, we simulate the four scenarios proposed by Costa. The simulations are similar to the previous experiment, with the difference that the new exogenous inputs are added to the initial demand. Negative input values circulate in the economy in the same

way as positive values. The agents split negative demands in smaller pieces until it requires a negative demand of R\$0.01 or more.

The results of the four scenarios are shown in Figure 1. There are some discrepancies in the estimated values, with the maximum absolute difference occurring in the profits. However, both results point out the same tendencies on every economic indicator, with maximum error found in scenario S_4 , whose average was 25% less than the analytical model. Therefore, this approach is capable of reproducing the same general behaviour of the analytical model as the mathematical meaning of the inverse of Leontief matches with the behavioural meaning of the presented agent-based model.

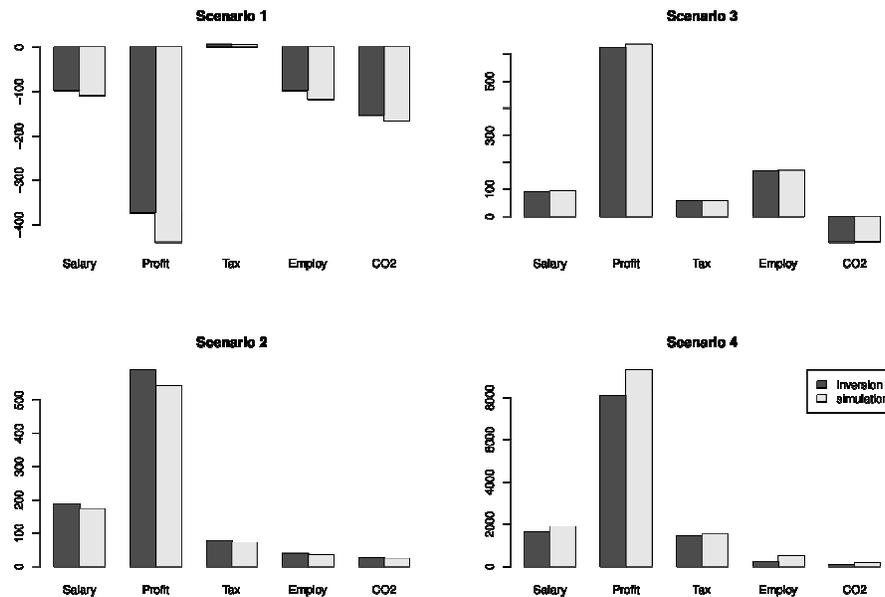


Figure 1: Differences between the simulated results and the analytical model.

6 An analysis of the model's assumptions

Based on the results presented above, we can now analyse the assumptions of this model. We focus on the discussion of three central points: *scenarios*, *behaviour*, and *space*. Of course, we are not arguing that the previous model (and consequently input-output analysis) is not

useful, but the objective is to discuss how we can enhance its confidence by relaxing assumptions.

The first assumption of the model is related to the scenarios, which has two main points:

Negative demands: Scenarios S_1 , S_3 , and S_4 propose that rural producers cut off their agricultural productions. This can be considered a bottom-up scenario because, although it is a top-down decision, the agents by themselves change their behaviour in order to accomplish such scenario, and the results should be generated based on this change on behaviour. However, the scenario is implemented as a completely top-down decision, by generating a negative demand, which circulates in the economy and reduces profit, salaries and the other indicators of the involved sectors. One question that arises when seeing these results is: because the analytical model is completely top-down, can it reproduce negative demands in a trustful way? In fact, applying negative demands is a *trick* to study bottom-up scenarios as if they were top-down.

Capability to react to changes: Factories and commerce sectors can adapt themselves to future changes on the demand with some effort. Mining can also adapt itself because the final product is below the surface. Of course, those sectors would require from just a few days to months to adapt themselves, finding new workers and increasing the productive capacity, but this time scale is clearly less than one year, which is the time scale of the scenarios. On the other hand, agriculture-based sectors usually have a productive cycle with a starting and an ending time that depend mostly on environmental conditions, such as the beginning of the rain season or the end of winter, and can be deeply affected by environmental changes. It makes the production of agricultural commodities not a straightforward result of the exogenous demand, as the other economic sectors can be.

From point related to the capacity of reacting to changes we can derive the second assumption of the model. Besides being completely reactive, the behaviour of every agent is also *homogeneous*. It means that the agents react in the same way to any demand. They have no threshold, phase transition, nor adaptation. As we can see previously, not every economic sector is capable of reacting in the same way to a given scenario. Only an agent-based approach can fulfil these requirements as it allows more flexibility to describe the behavioural

entities. With that, we can explore more realistic scenarios, with are based not only on changes in demand, but also changes in behaviour. For example, we can suppose that agricultural producers can fulfil only 50% of the increase in the exogenous demand. After that, another entity is be necessary to fulfil the demad, which would make the prices to rise. This approach opens up the possibility to investigate processes such as sinking, where there is a controlled spatial region, and the emissions may increase in another region as consequence of the unmet demand.

Finally, in the same way the model supposes that behaviour is homogeneous, it also supposes that geographic space is homogeneous. It means that the model supposes that the resources are equally distributed over space, and that carbon emissions and production are connected throuth a linear equation. Therefore, there is a need to add a spatial representation to the model using data available in the literature, for example deforestation [12] and biomass estimation [17]. Moreover, agricultural producers have a strong link with spatial locations. Their decisions are based in the land characteristics, from where they extract their productions. Therefore soil quality, rain quantity, slope and other characteristics affect the decisions of small as well as large farmers. Consequently, grounding agents into spatial locations.

In order to add a spatial representation to the model, we can break up the nodes of the model presented in Section 3 individually, generating societies of agents that represent, if not a one-by-one mapping of the primary producers, at least a representative set of the different behavioural trajectories applied to different small regions. As we can do it separately for each agent, it is possible to downscaling only the parts of the model that are object of study, instead of breaking up the whole economic chain.

7 Final remarks and future work

The idea of the model proposed on this paper is to calculate the inverse of Leontief matrix on-the-fly. Although the model is intuitive and there is not mathematical prove that it works, it is rather similar to the algorithm to calculate the inverse of Leontief proposed by [25], and the results of the simulations are similar to the ones using the inverse of Leontief. However, the agent-based paradigm give us more flexibility, opening up new possibilities for working with other types of economic scenarios and integrating it with other models.

The model presented in this paper produces a collection of agents based on input-output matrixes. From this initial model, we can create

models that do not rely on the assumptions and limitations of the input-output analysis:

1. Homogeneous behaviour and reactivity-based model;
2. Bottom-up scenarios implemented as if it was exogenous;
3. Scenarios limited to changes on the demand, not on the behaviour;
4. Spatial distribution of resources completely homogeneous.

The next steps of this model include adding a land use change component for the region. It will allow us to study the spatial effects of the natural resources and the evolution of the landscape within the scenarios presented in this paper, considering the local heterogeneities. We plan to use the idea of Generalized Proximity Matrix (GPM) [1] to establish the relations involving agents and/or spatial partitions (called *cells*), as proposed by [2]. The objective is to use three of the four presented relations:

agent → **cell**: agents execute land use decisions within their farms, which are represented by sets of cells.

agent → **agent**: commercial relations between agents are represented directly within them.

cell → **cell**: spatial neighborhood between cells can be used by the land use change process.

Within this new context, the model will better represent the spatial and behavioural heterogeneity of the processes taking place in the region. Consequently, we expect that it will bring more confident results to carbon emission scenarios for the Southeast Pará.

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