

EXERCISE ON SPATIAL PRISONER'S DILLEMA

Paper references

Nowak MA & Sigmund K (2000), Games on Grids. In: The Geometry of Ecological Interactions: Simplifying Spatial Complexity, eds. Dieckmann U, Law R & Metz JAJ, pp. 135–150. Cambridge University Press.

Lindgren & Nordahl (1994), Evolutionary dynamics of spatial games. *Physica D* 75 (1994) 292-309

We consider an infinitely repeated Prisoner's Dillema, where in every round each player has a choice between the two moves C (for cooperate), and D (for defect), and the pay-off matrix M has the following form:

	Player 2 Coop	Player 2 Defect
Player 1 Coop	(R,R)	(S,T)
Player 2 Defect	(T,S)	(P,P)

where $T > R > P > S$ and $2R > T + S$, where T (“temptation”), R (“reward”), P (“punishment”), and S (“sucker’s payoff”) are used to denote the results. These constraints imply that in a single game it is rational for both players to defect, but also that it would be to the mutual advantage of the players to establish cooperation in the long run. In the model, the moves of the players are influenced by noise, so that in each round an intended move is replaced by its opposite with probability p_{err} .

Each cell of a regular cellular space contains a single agent that plays a certain strategy in the game. Each agent interacts with himself and his four nearest neighbors on a 2D square lattice. Each agent plays the game against all neighbors, which gives each individual a total score. All agents live for a single generation and reproduce. The individual with the highest score in a neighborhood of a certain site gets to reproduce at that site.

A strategy is a rule for determining the next move of a player given the history of the game. There are strategies that have no memory, such as AC (always cooperate) and AD (always defect). Some strategies have a one-time memory, such as TFT (“tit for tat”) and WSLS (‘win-stay, lose-shift’). Tit-for-tat does whatever the opponent has done in the previous round. It will cooperate if the opponent has cooperated, and it will defect if the opponent has defected. WSLS repeats the previous move if the resulting payoff has met its aspiration level and changes otherwise. If the play on the current round resulted in a success (e.g., the payoff is either T or R), then the agent plays the same strategy on the next round. Alternatively, if the play resulted in a failure (e.g., the payoff is either P or S) the agent switches to another action.

In this exercise, you should implement the AC, AD, TFT, and WSLS strategies in a cellular space and find out what are the possible outcomes, considering the following payoffs:

- (a) T (“temptation”): $1 < T < 2$
- (b) R (“reward”): 1
- (c) P (“punishment”): $0 < P < 1$
- (d) S (“sucker’s payoff”): 0