EVALUATION OF THE PERFORMANCE OF FUZZY LOGIC APPLIED IN SPATIAL ANALYSIS FOR MINERAL PROSPECTING

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ABSTRACT Fuzzy logic, decision making procedure (AHP), and conditional probability were evaluated on the spatial analysis of geological data, to address potential areas for radioactive mineral occurrences in the Poços de Caldas Plateau ($\cong$ 750 Km^2). Spatial inference techniques were applied controlled by a prospecting model based on diagnostic criteria, represented by favorable lithology, structures features and gamma-ray intensity. The diagnostic criteria, favorable lithology and gamma-ray intensity, were converted on fuzzy members through linear functions applied over weights defined from a heuristic manner. In the lithological case linear functions were applied to spatialise the uncertainly associated with the location of the inferred boundaries. Structural features were spatialised through sigmoid functions that constitute the fuzzy members decreasingly ordered as the distance from the features increase. The crossover point of the sigmoid function was defined using the spatial correlation (conditional probability) between the diagnostic criteria and the 48 known radioactive mineral occurrences. Finally the fuzzy evidence was combined through a weighted mean sum where the weights were empirically defined according to the Analytical Hierarchic Process (AHP). The resulting grid was arbitrarily sliced in four classes of potentiality (Null, low, medium and high potential), which express different levels of favorability for radioactive mineral occurrences. The fuzzy scenario was qualitatively and quantitatively compared with previous results obtained by Boolean and Weighted Means based models. The comparison of both procedures showed that the fuzzy scenario accomplished a better performance. A target area of 30.43Km^2 (4.18% of the alkaline complex) encompassed 27 mineral occurrences, whereas each previous model encompassed only 24 in an approximately similar area. The correlation of mineral occurrences in relation to the potential classes for the different scenarios was also assessed. The high and medium potential classes of the fuzzy scenario showed posterior probabilities 12.9 and 5.7 higher than the prior probabilities. These values were, respectively, 12.6 and 4.97 for the same classes defined by the Weighted Means based model. For the potential class of the Boolean model this value was 5.78. These results show that fuzzy set theory enables a better refinement of the data modeling, improving the ability to spatially represent the geological data on continuous surfaces. This improvement can lead to better conclusions during decision making processes.

Keyword: spatial analysis, fuzzy logic, GIS and decision making process.

INTRODUCTION

Models of spatial analysis on geographic information systems (GIS) are normally simplified representations of natural phenomena. The core of this simplification is very often related to: a limited understanding of the phenomenon involved in the natural process; the complexity of the natural processes; and the limitations of the mathematical techniques used for representing these phenomena. The direct consequence of this simplification is normally the decreasing of variables involved in the modeling, which can generate imprecisions that results in misleading conclusions.

The genesis of mineral deposits is a good example for situations where the high complexity of physical and chemical processes can lead to an incomplete understanding of the deposit formation. In fact, Bonham-Carter (1994) mentioned that these processes are often very complex for a direct prediction of a mathematically-expressed theory, it makes it necessary that the prospecting process for mapping favorable areas rely mainly on empirical relations defined by "deposit models". These models consist of a large number of known deposits, considered to be sufficiently similar in terms of their characteristics, working as guides (description model) for prospecting new deposits of the same type. Therefore, in studies based on GIS, the "deposit models" play an important role both in the selection and derivation of the evidence, and in the definition of the weights that will be assigned to the evidence (Bonham-Carter, 1994).

On the other hand, although GIS applications are efficient tools, it makes it necessary that the prospecting model be executed by appropriate statistical analysis and adequate mathematical models (Turner & Sjoekri 1999). Following those principles, various authors (Harris 1989, Bonham-Carter 1990, Eastman 1995, Almeida-Filho 1995 and Burrough and McDonnell 1998) are interested in more precise and coherent modeling. They are searching for mathematical techniques which will allowed better representation of those processes involved in natural phenomena.

Another aspect that must be considered during spatial analysis built for mapping potentiality is the quality of the generated scenarios. Burrough & McDonnell (1998) mentioned that the quality of maps generated on GIS is evaluated, in the majority of the cases, only by their visual aspects. However assessments based only on visual aspects are insufficient, whether because the present information was wrong or because it was violated by errors during the computer procedure, as uncertain and errors are intrinsic to spatial data.

Being so, the present work intends to contribute to the study of techniques more fitted for spatial inference studies. Thus, fuzzy logic was evaluated in predicting potential areas of radioactive mineral occurrences in the alkaline complex of Poços de Caldas. This method was controlled by a prospecting model based on diagnostic criteria, represented by the occurrence of favorable lithologic units, structural features and radioactive anomalies. The result is a scenario of potentiality that targeted areas more suitable for radioactive mineral occurrences.

To assess the performance of the fuzzy scenario qualitative and quantitative studies were conducted, and its achievements were compared with former results obtain by Almeida-Filho (1995).

MATERIAL

The prospecting model in this work was developed on the SPRING package (Geo-referenced Information Processing System). A GIS based on a object-oriented data model, where its menus interface and programming language were derived (Câmara et al., 1996).

Study area

The study area encompasses the Poços de Caldas Plateau, which is located on the boundary between States of Minas Gerais and São Paulo, about 300 km from the city of São Paulo city (Figure 1a). The alkaline complex has a roughly circular shape with an area of approximately 750 Km^2, and a diameter of 35 Km.

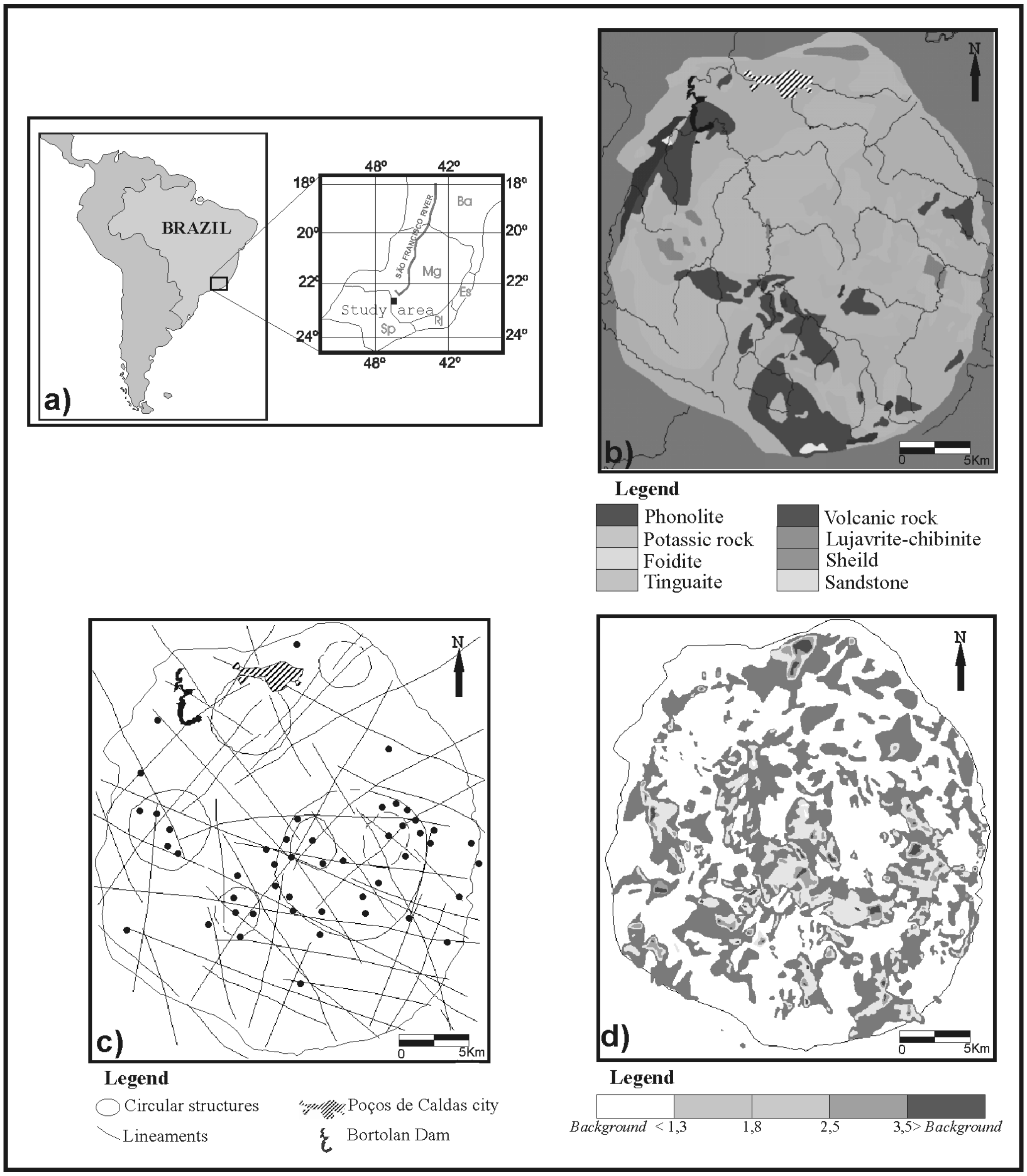

Figure 1 – Study area location (a); lithological map of the alkaline complex (Source: NUCLEBRÁS, 1975a) (b); structural map (Modified: Almeida-Filho, 1995) (c); and gamma-ray intensive anomaly map (Source: NUCLEBRÁS, 1975b) (d).

The alkaline complex, an intrusive body from the Mesozoic-Cenozoic age, is predominantly compounded of nepheline-syenite rocks (tinguaite, phonolite and foidite) (Figure 1b). The various lithological units, from an alkaline origin, can be divided into three main groups: breccias, tuffs and agglomerates; effusive and hypabyssal rocks; and plutonic rocks (Fraenkel et al. 1985). Volcanic rocks represented by breccia, tuff and agglomerate crop out in the northwest part of the massif. Effusive and hypabyssal rocks are formed by phonolite and tinguaite respectively and spread all over the alkaline complex. Plutonic rocks, consisting of foidite, lujavrite and in lower proportion chibinites, occur emplaced in effusive and hypabyssal rocks. Another important lithological unit of the massif is generated by hydrothermal and weathering alteration of the tinguaite. It is named "potassic rock", and is one of the most important lithological controller of radioactive minerals (Almeida-Filho 1995).

K-Ar Geo-chronological data (Bushee 1970) allow the estimation of the alkaline complex magmatic evolution 30 million years ago, where its first manifestation occurred in the Upper Cretaceous (87 million years ago). A study realized by Ellert (1959) recognized 6 stages of evolution in the complex formation: (a) shield ascending; (b) volcanic activity; (c) caldera formation; (d) alkaline magmatic activity; (e) annular dike formation; (f) foidite, chibinite, lujavrite intrusion.

Two large fault systems with predominant directions of N60W and N40E occur on the alkaline complex. The first one is correlated with the regional tectonic and the second with the caldera formation process (Fraenkel et al. 1985). Almeida-Filho & Paradella (1977), addressed through interpretation of the Multi-scanner (MSS) satellite image, the existence of 7 circular structures in the inner section of the alkaline massif, probably associated to former volcanic cones (Figure 1c). The occurrence of radioactive minerals all along the border of these structures, lead those authors to consider them as regional structural controllers of the mineralizations.

There are 48 known radioactive mineral occurrences (Figure 1c) that can be grouped in three associations: uranium-zirconium, thorium-rare earth and uranium-molybdenum (Tolbert 1966, Oliveira 1974 and Fraenkel et al.1985). The uranium-zirconium association is the most common mineralization, appearing as alluvial and eluvial deposits. The thorium-rare earth represents the second-best association, being the Morro do Ferro deposit its main representative. The uranium-molybdenum association is the most important one, occurring predominantly in lode strips or in lenticular bodies.

Prospecting model

Models of spatial inference and integration developed with GIS, rely mainly on empirical relationships, defined with the aid of the descriptive "deposit model". The strength of these models fundamentally depends on the previous geological knowledge of the study area, which will lead to the selection of the most important input data for the model.

In the present study, the complexity of the phenomena involved in the formation process of the alkaline complex and its mineral deposits, has made the conception of a empirical prospecting model that has considered every characteristic associated in the mineralization process very difficult. It involved tectonic, structural, lithological and weathered aspects that present particularities from one area to another. In spite of this matter, Almeida-Filho (1995) identified certain common characteristics of the radioactive mineral deposits, which were assumed as the control parameters of their occurrences, constituting the diagnostic criteria of the prospecting model:

- <u>favourable lithology</u>: occurrence of lithological controllers, represented by the potassic, volcanic rock and intrusive bodies of foidite;

- faults and fractures: appearance of faults and fractures, controlling the veins and lenticular bodies emplacements;
- gamma-ray total count: occurrence of total radioactive anomaly (higher than 1.8 times the background values) (Figure 1d).

Geographical database

To perform the study, a spatial database (Table 1) were constructed in the SPRING package, where the data were converted to UTM/SAD69 projection, using as cartographic reference the Poços de Caldas, Santa Rita de Caldas and Andrada sheets, with 1:50.000 scale, produced by Brazilian Statistical and Geographical Institute (IBGE) (Almeida-Filho, 1995).

Table 1 – Types, formats and data attributes

Data	Format	Attributes
City and main streams	Vector	Infra-structure
Lithological map	Vector and matrix	Lithological units
Phonolite intrusive bodies	Vector and matrix	Boundaries zones
Gamma-ray anomaly	Vector and matrix	Radioactivity (total counter)
Structural features	Vector	Faults, fractures and circular structures
Mineral occurrences	Vector	Field information

Modified from Almeida-Filho (1995).

MODELING

Fuzzy logic has been extensively used in spatial analysis developed by GIS (Burrough 1989, Burrough & Heuvelin 1992, Banai 1993 and Altman 1994). Fuzzy models overcome conventional models that force specialists to define crisply dicotomic rules with normally artificial boundaries that decrease the ability to efficiently organize solutions for complex problems, so common in natural processes. For some researchers (Zadeh 1972, Cox 1994, Fang 1997) the main benefit of models based on fuzzy logic is the capability for codifying knowledge, in certain way that approximates how specialists think in decision processes. The inference systems based on fuzzy logic allowed knowledge to be gained near the cognitive models used by experts during decision processes.

Mathematically fuzzy group is defined as follow: if Z denotes a space of objects, then a fuzzy group A in Z is the group represented by an ordinary pair: $A = (z, MF_A(z))$ for all $z \in Z$, where the function $MF_A(z)$ is known as the mapping graduation of the z members in A. The member "1" fits completely in the conjunct, whereas the member "0" doesn't belong to the conjunct. The function has to be set in such a way that this condition is respected. The most common functions are the linear and the sigmoid functions (Burrough & McDonnell 1998).

In the present study, the fuzzy members were addressed by the geological evidence through fuzzy functions and then combined together according to the adopted prospecting model. To map the fuzzy members of gamma-ray intensity, a linear function ($f(x)=0,0125*x$) was applied over the weights defined by Almeida-Filho (1995). The purpose was to rearrange the gamma anomaly classes to values ranging from 0 to 1.

In the lithological case, the procedure for converting the lithological units into fuzzy members also considered the semantic information of the lithological boundaries (inferred and defined). To accomplish that, thematic masks were built, bounding the transition zones of 100m defined along the inferred boundaries. They work as spatial controllers during the attribution of the lithological fuzzy members. The lithological evidence was edited in a manner, that each lithological unit was individualized into binary evidence (foidite binary evidence, lujavrite binary evidence, and so on). In conjunction theory, conjunct *A* would be confronted to *Not-A* ($\overline{A} = T - A$). Subsequently, distance maps were created adopting the boundaries of each lithological binary evidence as the starting point. They served as numerical base during application of the fuzzy member functions.

The next step consisted of constructing the fuzzy lithological evidence. They were obtained through the application of the fuzzy member functions on each of the distance maps, controlled by the masks and the 100m buffers along the boundaries. The buffers equal to a transition zone and serve to bound the region where the fuzzy members express the possibility of the inferred boundary location. The formulation below exemplifies the procedure applied on the potassic rocks:

> $MF_{potassic\ rock} = 0$ **IF** Thematic-Mask.Class = defined boundary **AND** Binary.evidence.Class = Not-potassic rock;
>
> $MF_{potassic\ rock} = 1$ **IF** Mask.Class = defined boundary **AND** Binary.Evidence.Class = Potassic rock;
>
> $MF_{potassic\ rock} = (0.005 * dist) + 0.5$ **IF** Thematic.Mask.Class = inferred boundary AND Binary.Evidence.Class = Potassic rock **AND** distance < 100m;
>
> $MF_{potassic\ rock} = (-0.005 * dist) + 0.5$ **IF** Thematic.Mask.Class = inferred boundary **AND** Binary.Evidence.Class = Not-potassic rock **AND** Distance < 100m.

These formulas ensure that the original boundary location meshes with the crossover point MF=0.5. Points that lie on the lithological units and on the transition zone assume values from 1 to 0.5. Outside the lithological unit but in the transition zone the values drop from 0.5 to 0 according to the distance.

The Figure 2 illustrates the result of the procedure applied over an inferred boundary between potassic and volcanic rocks. In the left part of the figure are represented the fuzzy members of the potassic rock, and in the right part the fuzzy members of the volcanic rock. The dotted lines delimit the transition zone of 100m between the two units. The fuzzy member function that graduated the elements are shown at the top of the figure.

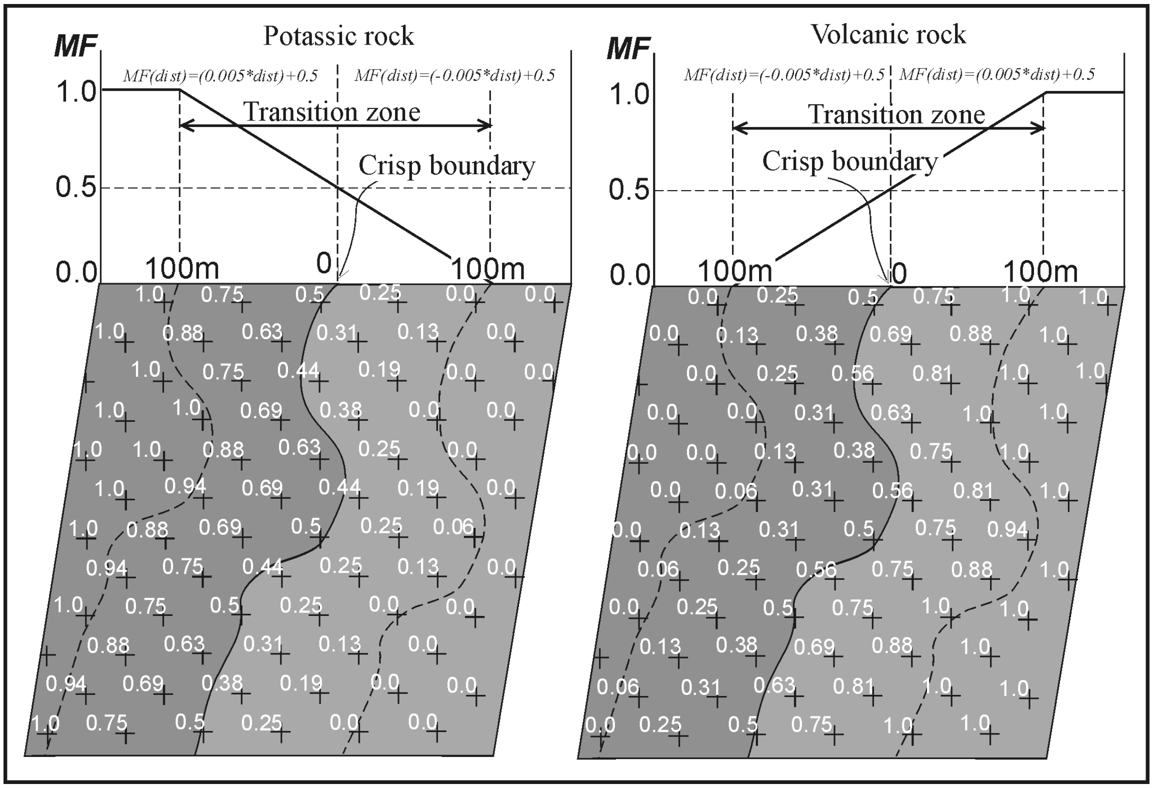

Figure 2 – Exempla of fuzzy members on a transition zone between the lithological units, potassic rock (dark gray) and volcanic rock (light gray).

The fuzzy lithological binary evidence was combined through a weighted means sum (Equation below). The weights, empirically chosen, intended to express the importance grade of each lithological unit to the adopted prospecting model.

$$MF_{litho\log ical} = potassic\ rock + (lujavrite, chibinite) + (0.5 * foidite) + (0.33 * volcanic\ rock)$$

The procedure of attributing fuzzy members to the circular structures and lineaments were similar. Firstly, two distance maps were built considering the structural features as the starting point. The sigmoid functions, which spatialised the fuzzy members, were applied on the distance maps considering transition zones of 700m and 500m respectively. They spatialised the members in a gradual decreasing form as the distance from the structural features increased. The Figure 3 illustrates the fuzzy member function of the lineaments.

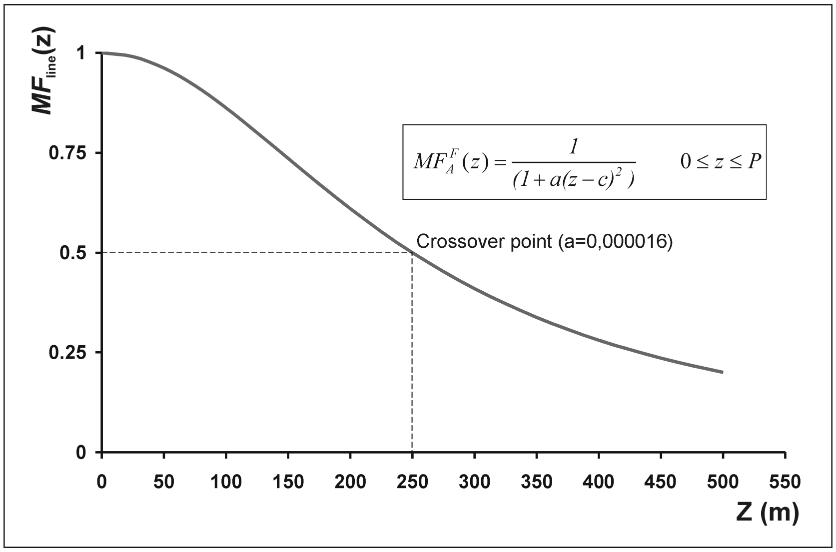

Figure 3 – Sigmoid function of the lineament fuzzy members.

The process to spatialise the phonolite intrusive bodies was similar to the structures features' procedure. Once again a sigmoid fuzzy function modeled the importance of these bodies regulated by the distance from the boundaries of the intrusive bodies.

In the inference the evidence was combined through a weighted sum. The weights of each evidence was empirically defined through the Analytical Hierarchic Process (Saaty, 1992). This technique allows definition through a pairwise comparison of a hierarchical relationship between the fuzzy members. The hierarchy as a structural form of a system allows expression of complex problems through both the decomposition of its elements and through the identification of the different classes that comprehend the whole system.

The logic of the pairwise comparison suggests to obtain a relative value of the merit in situations where there are any uncertainty about the criteria to determine the desired patterns in a spatial analysis procedure. The pairwise comparison is a decomposed analysis produced by pairs of elements. This analysis concludes with a synthesis of recomposition through the aggregation of the member value, in a method of unified assessment (Banai 1993).

First the fuzzy evidence was evaluated in pairs. This assessment attempted to capture, through the knowledge of the specialist, the relative importance level between the evidence. The spatial analytical package of the SPRING (AHP decision support) allows graduation to the ninth level (equal, slightly better, little better, moderately better, better, quite a bit better, much better, critically better, utterly better) that were utilized as input data of a pairwise comparison matrix. The considered relationships are listed below and the matrix is showed in Table 2:

Gamma is little better than lithology (3 : 1);
Gamma is better than circular structure (5 : 1);
Gamma is much better than Lineaments (7 : 1);
Gamma is much better than the phonolite intrusive bodies (7 : 1);
Lithology is slightly better than circular structure (3 : 1);
Lithology is better than lineaments (5 : 1);
Lithology is better than the phonolite intrusive bodies (5 : 1);
Circular structure is slightly better than lineaments (3 : 1);
Circular structure is slightly better than the phonolite intrusive bodies (3 : 1);
Lineaments are equal to the phonolite intrusive bodies (1 : 1).

Table 2 – Pairwise comparing matrix

	Gamma-ray anomaly	Lithology	Circular structure	Lineaments	Phonolite intrusive bodies
Gamma-ray anomaly	1				
Lithology	1/3	1			
Circular structure	1/5	1/3	1		
Lineaments	1/7	1/5	1/3	1	
Phonolite intrusive bodies	1/7	1/5	1/3	1	1

The module measures the autovectors of the matrix, that corresponds to the evidence's weights and the consistence grade, which indicates the coherence of the defined relations. The consistence grade obtained was 0.03, which indicates a good agreement between the comparisons. The weights obtained for each fuzzy evidence was:

Gamma-ray anomaly: 0.514;
Lithology : 0,258;
Circular structure : 0.122;
Lineaments : 0.0529;
Phonolite intrusive bodies : 0.0529.

The fuzzy evidence was combined through a weighted mean sum, as is shown below:

$$MF_{result} = (MF_{gamma} * 0{,}514) + (MF_{litho\log y} * 0{,}258) + (MF_{circular\ struct.} * 0{,}1223) + (MF_{lineaments} * 0{,}0529) + (MF_{\text{int}\,rusive\ bodies} * 0{,}0529)$$

The final result is a GRID that expresses, in each grid location, the potentiality for radioactive mineral occurrences. This GRID was finally sliced in arbitrary classes of potentiality (null, low, medium and high potentiality), in a manner to identify the target areas with higher potentiality.

RESULTS

The potential areas defined by the fuzzy spatial inference were assessed qualitatively and quantitatively, where the results compared with former results generated by Almeida-Filho (1995). In the qualitative analysis the number of the 48 mineral deposits mapped by each of the potential classes was counted. The high potential class encompassed 12 mineral occurrences. This number rises to 27 (56% of the deposits) when the medium potential class is also accounted for (Joined area = 30,43 Km^2 or 4,18% of the alkaline massif). On the other hand, the methods applied by Almeida-Filho (1995) mapped 24 deposits with both the favorable class of the Boolean based model (Area = 32,4 Km^2), and the joined classes (High and Medium classes) of the Weighted Means based model (Area = 30,64 Km^2). This result shows that the fuzzy scenario was more efficient, mapping 3 deposits more in an area of equivalent surface ($\cong$ 30Km^2).

In the quantitative analysis conditional probability was employed to assess the correlation between the scenarios' potential classes and the mineral occurrences. To accomplish that a parameter called "confident grade" (Equation below) was applied to measure the correlation between the potential classes and the 48 deposits. The adopted prospecting model shows coherence if the high potential classes have a positive correlation to the deposits present.

$$Confident\ grade = \frac{P(deposit \mid potential\ class)}{P(deposit)}$$

The confident grade showed that both the fuzzy scenario and the ones generated by Almeida-Filho (1995) yielded coherent results. The low potential classes obtained small values which indicate a negative correlation whereas the high potential classes attained greater values that represent a positive correlation.

However the confident grade values of the high (12.9) and medium (5.7) potential classes of the fuzzy scenario were higher than those achieved by the correspondent classes of the weighted means based model, 12.6 and 4.97 respectively. The favorable class of the

Boolean based model reached a confident grade of 5.78. Table 3 summarizes the obtained results.

Table 3 – Summary of results obtained by the scenarios of potentiality for radioactive mineral occurrences

		Boolean*	**Weight Means**	**Fuzzy**
High potential class	Area (km^2)	32,37	6,48	6,14
	Confident grade	5,78	12,6	12,9
	Deposits	24	12	12
Medium potential class	Area (km^2)	-	24,16	24,29
	Confident grade	-	4,97	5,7
	Deposits	-	12	15
High + Medium potential classes	Area (km^2)	32,37	30,64	30,43
	Deposits	24	24	27

* - results referred to the favorable class.

CONCLUSION

The fuzzy logic proved to be a powerful technique on spatial analysis of geological data. However, the success of such an approach relied directly on both the level of knowledge of Poços de Caldas' geology and to the more suitable manner of how to spatially represent them. The fuzzy logic allowed the representation of the spatial variation of the geological evidence on continuos surfaces. In addition, the semantic import approach (IS) easily embodied the uncertainty of the location of the inferred lithological boundaries. Those approaches permitted the obtaining of a more accurate model.

The AHP technique was also directly responsible for the good performance of the fuzzy scenario. It efficiently hierarchized the geological evidence according to the relative importance adopted on the prospecting model.

The quantitative analysis allied with visual checks strengthened the above deductions, where the fuzzy scenario showed a slightly superior performance compared to the ones achieved by both Boolean and weighted means based models. Therefore it can be concluded that the fuzzy logic is more efficient for studying natural phenomena.

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