GAME THEORY AND AGENT-BASED MODELLING FOR THE SIMULATION OF SPATIAL PHENOMENA

Pedro Ribeiro de Andrade

Doctoral Thesis in Applied Computer Science, advised by Dr. Antônio Miguel Vieira Monteiro and Dr. Gilberto Câmara

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Dr. Lamartine Nogueira Frutoso Guimarães

_____
Presidente
IEAv, São José dos Campos (SP)

Dr. Antônio Miguel Vieira Monteiro

_____
Orientador
INPE, São José dos Campos (SP)

Dr. Gilberto Câmara

_____
Orientador
INPE, São José dos Campos (SP)

Dr. Lubia Vinhas

_____
Membro da Banca
INPE, São José dos Campos (SP)

Dr. Jaime Simão Sichman

_____
Convidado
USP, São Paulo (SP)

Dr. Antônio Carlos da Rocha Costa

_____
Convidado
FURG, Rio Grande (RS)

Dr. Tiago Garcia de Senna Carneiro

_____
Convidado
UFOP, Ouro Preto (MG)

Aluno: Pedro Ribeiro de Andrade

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ABSTRACT

Agent-based modelling (ABM) provides a bottom-up method for building complex systems. It has been gaining growing acceptance in various fields of science, such as economics, sociology, and anthropology, because of its capacity to show how collective social actions emerge from individual behaviour. One of the important research challenges for ABM concerns how to represent human actions in a landscape. The agents have to be grounded to a representation of the world, on which they can sense and act. In these studies, ABM toolkits need to support different types of geospatial data. However, the demands and formalisms for creating agent-based models for geospatial problems are not well established in the literature. This work presents advances on methodologies to represent and develop agent-based models of geospatial phenomena. First, it describes an architecture for studying geospatial phenomena using an agent-based approach. Second, it presents a couple of models that uses game theory as basis for the behaviour of agents. Agents compete for space, and their interactions are expressed as non-cooperative games. A definition of mobility is presented, and the effect of this and other parameters on the development of the models are investigated.

TEORIA DE JOGOS E MODELAGEM BASEADA EM AGENTES PARA A SIMULAÇÃO DE FENÔMENOS ESPACIAIS

RESUMO

A modelagem baseada em agentes (*Agent-Based Modelling* – ABM) fornece uma metodologia *bottom-up* para a construção sistemas complexos. Ela tem obtido uma crescente aceitação em vários campos da ciência, tais como economia, sociologia e antropologia, por causa de sua capacidade de expressar como ações coletivas podem emergir a partir de um comportamento social individual. Um dos desafios importantes na modelagem baseada em agentes diz respeito a como representar as ações humanas em um ambiente espacialmente explícito. Agentes têm de ser conectados a uma representação do mundo, que pode ser observada e sobre a qual eles podem tomar decisões. Para estes estudos, ferramentas para modelagem baseada em agentes precisam ser capazes de manipular os diferentes tipos de dados geoespaciais. Entretanto, as demandas e os formalismos para se tratar estes problemas ainda não estão bem estabelecidos na literatura. Este trabalho apresenta avanços em metodologias para se representar e desenvolver modelos baseados em agentes de fenômenos geoespaciais. Primeiro, é apresentada uma arquitetura para estudar fenômenos geográficos usando a abordagem baseada em agentes. Segundo, é apresentado um conjunto de modelos que usam a teoria de jogos como base para o comportamento dos agentes. Nestes modelos, os agentes competem por espaço, e a interação entre eles é expressa através de jogos não-cooperativos. Uma definição de mobilidade é apresentada e seus efeitos sobre o desenvolvimento dos modelos são investigados.

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1 INTRODUCTION

In the book *The Theory of Games and Economic Behavior*, John von Neumann and Oskar Morgenstern develop the first steps of the mathematical theory of games (VON NEUMANN; MORGENSTERN, 1944). Initially, the theory focused mainly on economics and on strategic games, such as chess, with the main formal contribution being the minimax algorithm for two-person, zero-sum game with finite strategies. At that time, there was a great discussion about whether or not the mathematical method was able to be applied for studying behaviour. Decades have passed by, and game theory evolved to a large framework that has been explored by different areas of knowledge, such as biology (MAYNARD SMITH, 1982), political science (OSTROM, 1990), and social sciences (KOLLOCK, 1994), currently being proposed as a universal language to unify all behavioural sciences (GINTIS, 2000).

In spite of the great success of game theory in the last decades, results of game theoretical models are highly dependent on their underlying assumptions, which are not necessarily true; sometimes they are believed as true due to lack of knowledge, but most of the times it happens because of the necessity to guarantee tractability of the mathematical approach. Traditionally, formal approaches for modelling social, economic, and ecological phenomena usually have strong assumptions such as homogeneity and rationality, with the main objective being to find out equilibrium points, also called stable states of the system. However, some works have shown that these assumptions do not stand in many cases, mainly the idea of rationality. There are some strong justifications against it:

a) People may try to be rational, but they can rarely meet the requirements of information and foresight imposed by rational models (SIMON, 1955);

b) Such a machine that could determine a rational solution for a given decision making process may not exist (BRIAN ARTHUR, 1994);

c) Some equilibrium points, although stable, may not even be attainable (EPSTEIN; HAMMOND, 2002).

Another dimension usually oversimplified, or even ignored, is *space*, mainly due to the difficulty to be handled. Social, economic, and ecological models of dynamic processes are examples of areas where space is a fundamental source of heterogeneity, affecting the way by which people interact, with properties such as anisotropy,

non-stationarity, and cyclic or rare events, which can clearly affect the outcomes. Moreover, even if one argue that newest communication technologies promise to ignore the impact of distance on human relations (CAIRNCROSS, 1997), it has been shown that spatial proximity still remains crucial in effective collaboration, and the technological development mitigates only part of the negative impacts of distance (KIESLER; CUMMINGS, 2002).

Given all these restrictions of analytical models, and based on the fact that science evolves through the development of more and more complex models that can help us to improve our capacity to understand the reality, the only way to overcome these assumptions is by studying the underlying processes and their consequences through computer simulations. Computers can be used as laboratories for testing new behavioural theories by using artificial representations. Because of that, this approach has been seen as the third way of doing science, in contrast with the two standard methods of induction and deduction (AXELROD, 1997a).

Within the simulation approaches, agent-based modelling (ABM)[1] comes up as a promising alternative. ABM provides a bottom-up method for building complex systems, through the representation of individual entities and their dynamical interactions. It is a relatively new computational modelling paradigm and, within the last years, this research field has expanded far beyond its earlier boundaries in distributed artificial intelligence. Gilbert points out the main advantages of ABM:

> It allows modellers to represent in a natural way multiple scales of analysis, the emergence of structures at the macro or societal level from individual action, and various kinds of adaptation and learning, none of which is easy to do with other modelling approaches (GILBERT, 2008).

It makes this approach flexible enough to relax the assumptions required by traditional game theoretical models. Given that, it is possible to not only have more realistic models, but also to test new hypothesis that arise from this environment.

Consequently, game theory needs to be studied facing this new modelling para-

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[1]In this text, we use the term ABM following (HARE; DEADMAN, 2004), who distinguish "agent-based modelling" from "multi-agent simulation" in the sense that the former has been used for *artificial life agents*, and the later has been used for *software agents* (task-specific agents, entertainment agents, viruses). Also, the term ABM has a broader use in social sciences (AXELROD, 1997b; BANKES, 2002) and economy (BRIAN ARTHUR, 2006). The classification of "software agents" and "artificial life agents" was initially proposed by (FRANKLIN; GRAESSER, 1996).

digm. Traditional models have to be adapted or extended in order to analyse the spatial effects through an agent-based point of view. In order to accomplish that, it is necessary to evolve not only in the development of models, but also on the development of methodologies over which these models can be constructed, that is, building blocks that can be used by different agent-based models of geospatial phenomena.

The central objective of this thesis is to study how the general theory of equilibrium points, in this case Nash equilibrium and Evolutionary Stable Strategies (ESS), behave within a spatial environment that restricts the interactions between agents. In order to accomplish that, the models presented on this thesis relax some assumptions of analytical models but add two new assumptions: *limited resources* and *mobility*. The main questions this work analyses are: *Are the analytical equilibrium points the best reference for a competition for space? Is a population capable to evolve to such point?*

This thesis is written as a collection of papers. Each paper investigates a specific scientific question, but they are connected due to the necessity of having a spatially explicit representation within the scope of agent-based modelling. The thesis can be splitted into two parts: *representation* and *behaviour*.

The problem of representation relies on the challenge of having a framework that is capable of having abstractions in order to support any kind of spatial entities and their relations. Newest software toolkits have made ABM easy enough to be attractive to areas such as economics, sociology, anthropology, physics, and biology. The use of simulation toolkits relieves the modeller of programming the parts of the simulation that are not content-specific, such as simulation control and data structures (CASTLE; CROOKS, 2006; RAILSBACK *et al.*, 2006; TOBIAS; HOFMANN, 2004). It also increases the reliability and efficiency of the model, as the most complex parts have been created and optimised by professional developers. However, the demands and formalisms for creating agent-based models at this complexity level are not well established in the literature. Because of this, newest toolkits, as well as spatial extensions of older toolkits, have been created according to the class of problems they are trying to support, instead of following a more conceptual and general approach. Two chapters tackle this problem:

Chapter 2: The objective of this chapter is to present a contribution towards generalising the representation of geospatial entities and their relations for

simulating complex spatial systems using the agent-based approach. We argue that any general toolkit should be able to represent the four types of relation explicitly. These relations can be grouped in two classes, placements and neighbourhoods, with likenesses and differences between them. Given that, we define requirements for representing geospatial entities and their relations.

Chapter 3: Following the requirements described on Chapter 2, this chapter describes a general architecture for building and simulating agent-based models that use real-world geospatial data, taking into account all the ways geospatial data can feed these models. Although the approach focus on real data, the architecture can also work with artificial environments.

The second part of this thesis use part of the proposed architecture presented on Chapter 3 to focus on spatial behaviour, materialized as mobility decisions. Commonly in the literature, decisions related to spatial mobility are taken by a straightforward reaction over some observation on the spatial configuration, or acted as an opportunity. This thesis broadens this view by exploring decisions that are based on the results of the interactions among agents, using a game theoretical approach. Two chapters are related to this problem:

Chapter 4: This chapter describes a new model for spatial games called *games on cellular spaces*. We present a definition of mobility in terms of the satisfaction of an agent with its current spatial location. Agents compete for space through a non-cooperative game by using mixed strategies. We are particularly interested in studying the relation between the theoretical definition of Nash equilibrium and the winner strategy of a given model, and how mobility can affect the results. Instead of working with the idea of rationality, we use populations of agents that cannot change their strategies.

Chapter 5: Based on the concept of games on cellular spaces developed on Chapter 4, this chapter studies the dynamics of populations competing for space through evolutionary rules. We study whether a population is capable to evolve and adapt itself to a stable state, and how the attained equilibrium is related to the theoretical equilibrium. Additionally, it analyses the effects of each single parameter in the development of the model, investigating their effects on the equilibrium state, mainly when they go towards the analytical assumption.

2 ENTITIES AND RELATIONS FOR AGENT-BASED MODELLING OF COMPLEX SPATIAL SYSTEMS[1]

Abstract

This work presents a contribution towards generalising the representation of geospatial entities and their relations for simulating complex spatial systems using the agent-based approach. We analyse the literature, and argue that each of the four types of relation is necessary. These relations can be grouped in two classes, placements and neighbourhoods, with likenesses and differences between them. Given that, we define requirements for representing geospatial entities and their relations, and study seven agent-based modelling toolkits (NetLogo, OBEUS, Repast, Swarm, Cormas, GRSP, and TerraME), analysing their capabilities to address the proposed requirements. Finally, we present our current work and future directions on developing TerraME.

2.1 Introduction

The human being has always changed its environment. Changes can occur at the physical, social, and economic levels, ranging from deforestation to global warming, from individual mobility to segregation patterns, or from an ordinary bargaining between buyer and seller to an economic chain dynamics, to name a few. They result from complex interactions among the entities directly and indirectly involved in the system. The inherent complexity of the processes involved in the system shows non-linearity, displayed by a complex behaviour that presents thresholds, positive and negative feedback loops, lags in time and space, resilience, heterogeneity, and, above all, surprises (LIU *et al.*, 2007).

The living earth is the natural support where complex relations evolve over time. It has great influence in our decision-making processes, and in how we set up and

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[1]This chapter is an extended and slightly revised version of the paper co-authored with A. M. V. Monteiro and G. Camara, *"Entities and Relations for Agent-based Modelling of Complex Spatial Systems"*, Proceedings of the First Brazilian Workshop on Social Simulation (BWSS), 52-63, 2008.

maintain relations, because we are grounded to places and regions with unique characteristics and dynamics. Although some production factors such as capital, labour, or even information are less spatially sensitive (at least in developed places), natural resources and consumers are where they are. Moreover, even though the newest communication technologies promise to ignore the impact of distance on human relations (CAIRNCROSS, 1997), proximity remains crucial in effective collaboration, and the technological development mitigates only part of the negative impacts of distance on productivity (KIESLER; CUMMINGS, 2002).

With the development of technologies for acquisition and manipulation of data, especially geospatial data, it is now possible to work with models at individual level. Different methods for data acquisition are available, such as GPS and remotely sensed data, surveys, participant observation, and field and laboratory experiments. These methods can be combined to increase the model effectiveness (ROBINSON *et al.*, 2007), but the main difficulty stands in how to manipulate the diversity of data provided by these methods.

In the past, researchers have used Geographic Information Systems (GIS) to study these interactions. But, although GIS have strong functionalities to manipulate and to analyse geographical data, it is hard to model temporal dynamics at individual level within GIS environments because they are based on data models, instead of process models (RAPER; LIVINGSTONE, 1995). Usually, GIS with embedded dynamical predictive models are built as customised tools and work only with a specific process (BATTY, 2005). In fact, most of them are created with physical rather than human processes in mind. The only way to model social or economic problems is thinking in human behaviour as if it was physical. For example, applications that use Cellular Automata (CA) for studying human behaviour anthropomorphise the state variables of a cell.

Agent-Based Modelling (ABM) comes as an approach for addressing these issues broadly. ABM provides a bottom-up method for studying complex systems through the dynamical interaction of agents. It allows representing individual behaviour and cognitive process, which cannot be fully attained by approaches such as mathematics or statistics. Although some disbelief has taken place on this approach (COUCLELIS, 2001; JONES, 2007; LEOMBRUNIA; RICHIARDI, 2005), it has been shown that ABM can unquestionably increase our understanding of human behaviour (BANKES, 2002; BOUSQUET; LE PAGE, 2004; BRIAN ARTHUR, 2006; EPSTEIN, 2006).

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Newest software toolkits have made ABM easy enough to be attractive to areas such as economics, sociology, anthropology, physics, and biology. The use of simulation toolkits relieves the modeller of programming the parts of the simulation that are not content-specific, such as simulation control and data structures (CASTLE; CROOKS, 2006; RAILSBACK et al., 2006; TOBIAS; HOFMANN, 2004). It also increases the reliability and efficiency of the model, as the most complex parts have been created and optimised by professional developers.

Recently, agent-based models have begun to use real-world spatial data for studying complex spatial systems (BROWN et al., 2005). However, most of the toolkits were initially created to work with landscapes that do not consider the environment in which we move and interact. They rely on non-grounded agents, in the sense that they do not represent entities which can sense and act in a representation of the real world. To follow the current development, toolkits have to carry out the support for working with spatial data.

This work presents a contribution towards generalising the representation of geospatial entities and their relations for simulating complex spatial systems using the agent-based approach. We revise and analyse the works in the literature, and define requirements for representing geospatial entities and their relations. We focus on the geographic needs; for general requisites there is already a couple of works (GASSER, 2000; MARIETTO et al., 2003; RAILSBACK et al., 2006; TOBIAS; HOFMANN, 2004). We analyse some toolkits for ABM and their abilities to address the proposed requirements. Finally, we present our current work, towards fulfilling these requirements in the TerraME toolkit.

2.2 Reviewing the entities and their relations

In this section we present the works in the literature about the relations between the entities of agent-based models for complex spatial systems. Before defining their relations, we need to introduce the entities of a system, which belong to a *scale*.

2.2.1 Scale and its components

Gibson and others review the idea of scale in the literature of social science, especially related to the human dimensions of global environmental change (GIBSON et al., 2000). The causes and outcomes of the human actions can be measured at different levels along multiple scales. Scale is a well-defined idea in the natural sciences, but

it has fuzzy borders in the social sciences. Nevertheless, the authors show that each scale has three dimensions: spatial, temporal, and behavioural (analytical). In this work, we are interested in computer simulations, therefore each dimension has to be discretised in some way.

2.2.1.1 The spatial component of a scale

Spatial data has to be divided into single partitions, called *cells*, whose internal content is homogeneous, in such a way that any change in a cell affects all its extension. It is possible to change each cell independently from the others, which allows working with heterogeneous data. Dividing space in rectangular entities has advantages over the vectorial representation because it simplifies the representation and helps creating neighbourhood relations (TOBLER, 1979). However, vector data can bring more reality to the models, mainly when it represents some economic, political, or social convention.

2.2.1.2 The behavioural component of a scale

Instead of using a set of equations for describing the behaviour of a model in an aggregated way, the behavioural entities are described as *agents*. Each agent can affect itself, the spatial partitions, and the other agents, and the interactions between them can produce an emergent pattern. Modelling with agents allows a more aggressive exploration of characteristics such as imperfect rationality, social and institutional structures, and distinct behaviours in different locations.

Agents can represent geospatial entities such as farmers or householders. Therefore, just like for representing space, they can be associated to objects stored in a geographic database, whose information layer contains the spatial representation of the agents as well as their properties. Although there are some advances in visual languages, the behaviour of agents and a complementary part of their properties still have to be described in a programming language outside the GIS environment.

2.2.1.3 The temporal component of a scale

Each entity of a model can have a frequency in which it decides its actions. There are two choices for describing temporal events. The first is synchronous (parallel) execution, where each agent is an independent process in the simulation. The advantage of this approach is that the time one agent takes for its decision-making

is always considered. Thus, agents that need more time to compute their decisions are in disadvantage. The main problem of this approach is that the modeller has to give some attention to avoid temporal conflicts, which can significantly increase the complexity of the model.

The second alternative is asynchronous (sequential) execution, with the time discretised as a scheduler of actions. As an agent usually takes much more time to perform changes in the geographic space than to compute its decisions, we can usually ignore the processing time. Discrete time allows a straightforward solution to any temporal conflict between agents, defining a predetermined sequence of events to take place along the execution. As each agent has its own frequency of execution, it is possible to have events with different temporal frequencies, or even events occurring only once. The capacity to schedule events at different times allows the modeller to simulate the temporal particularities of the specific geospatial process under study.

In each step of its execution, a model is at a given time, which always increases. Once the scheduler finishes all the events of a given time, it increments the time and then starts processing the events of the next time. It makes the temporal component conceptually different from the other two because their extents are completely available at each time step. Therefore, *agent* and *cell* are the two main entities of a spatial phenomenon at a given scale, while the time is represented by the execution flow.

2.2.2 Relations following Torrens and Benenson

A relation is a mapping that describes the connection between two entities. Combining the two classes of entities, agents and cells, we have four types of relations: cell→cell, agent→cell, agent→agent and cell→agent. Torrens and Benenson use a leader and follower approach to represent these relations (TORRENS; BENENSON, 2005). The leader is responsible for managing the relation, while the follower is a passive object. The aim of the authors is to keep consistency in the relations, avoiding conflicts between the entities:

> [...] an application of the [...] rules that describe these changes [on the relations] might cause conflicts, when, in housing applications, for example, a landlord wants to sell his property, while the tenant does not want to leave the apartment. Who has the right to destroy the relationship between the tenant and the property, then? This example represents the general problem of consistency in managing relationships (TORRENS; BENENSON, 2005).

TABLE 2.1 - Cell and agent relations.

To From	Cell	Agent
Cell	Neighbourhood relation (static)	Agents who are in a certain cell (backward query)
Agent	Cells an agent controls (dynamic)	Prohibited relation (transition)

Source: (TORRENS; BENENSON, 2005).

Following the leader and follower idea, we have agents as leaders and cells as followers. The relation cell→cell has two followers; therefore it is a static representation, storing spatial neighbourhoods. On the other side, agent→cell specifies which cells one agent has or controls. Agents are free to manage this relation.

The two other relations do not fit in the leader and follower definition. As one cell cannot be a leader in a cell→agent, the authors solve it by using a backward query over agent→cell. Agent→agent is not feasible because it is not possible to define the leader of the relation. The authors argue that the only way two agents can communicate is by using the other available relations in a transitive way (agent→cell, then cell→cell, then cell→agent). Moreover, Benenson and Harbash argue that "this limitation might be inconvenient if you develop psychological model, but, luckily, we model not love affairs, but the collective urban spatial phenomena" (BENENSON; HARBASH, 2004). Table 2.1 summarises the four relations and how the authors solve them. The authors call the relations between two entities of the same type (agent→agent and cell→cell) as neighbourhood, and the other two as geo-referencing. They consider the relations as being binary, that is, at each time step, two entities are either related or not.

2.3 A critical analysis of the leader and follower approach and a new proposal

Torrens and Benenson cite an example of urban conflict, to which the leader and follower approach can solve the consistency problem shown in the last section:

> Accounting for limitations of OBEUS (direct relationships between agents are not allowed) the way to force the tenant to leave the property is to raise their payment. The tenant (the leader in tenant-property relationship) will likely end the relationship by herself in that case (TORRENS; BENENSON, 2005).

Even accounting the limits of this approach, the authors argue that

> [t]here is no proof that the majority of real-world situations can be imitated by the leader-follower pattern, although we are not aware of any natural instance where this pattern is insufficient (TORRENS; BENENSON, 2005).

The point on their example is that, in a real world situation, the landowner cannot one-sidedly increase the payment to force the tenant to leave, because this relation is often formally settled by a location contract. The only legal way to the landowner force the tenant to leave the property is by breaking the contract, accepting the outcomes imposed by its clauses. For that, we would need another agent with the power to break the relation, even without the tenant agreement. Therefore, the leader and follower approach cannot be used in this example.

Even if we consider true the statement that raising the payment will force the tenant to leave the property, we would need to assume that the seller owns all the rights on the contract, except the right to cancel it. But any toolkit that intends to have a generalised representation of the spatial entities and their relations cannot be limited to any *a priori* statement. Therefore, guaranteeing consistency in the relations has to be part of the model, instead of a restriction of the toolkit, because there can be another model that needs a different consistency in the relations.

The proposal of having agent→agent relations only in a strictly transitive way is another limitation of using the leader and follower approach. Patterns of change may arise locally, but social structures do not have to, and the technological development makes it even more fragmented. In fact, there are relations which do not rely on geographical closeness.

Instead of trying to prove that this or that relation is not necessary in a modelling toolkit, we propose that every relation type is necessary. Whenever there is a conflict over changing or not some relation, it has to be defined by the model. The ways to solve conflicts can be based on the idea of autonomy proposed by Barber and Martin (BARBER; MARTIN, 1999). They argue that:

> [a]n agent's degree of autonomy [...] is the degree to which the decision-making process, used to determine how that goal [of the agent] should be pursued, is free from intervention by any other agent (BARBER; MARTIN, 1999).

Conflicts can be solved by an agreement, when the agents follow a true consensus, or by a superior instance with more autonomy, called supervisor. One agent is locally autonomous only when no other agent can affect its decision-making. Barber and Martin use the term locally autonomous instead of just autonomous because they consider one agent may have different levels of autonomy for different goals (BARBER; MARTIN, 1999).

We divide the relations in two groups: *placements* and *neighbourhoods.* Following we describe each one, expose the reasons of this classification, and argue that each relation is necessary, presenting examples of works that identify or use them.

2.3.1 Placement relations

Each relation involving two entities of distinct types is a placement. The relations agent→cell and cell→agent represent the relations of this group. The first relation, cell→agent, stores the agents that belong to a cell. Each cell may have one or more agents within it, and one agent can be at most in one place at any given time. One can easily implement movement on space by changing this relation. This is the most common representation available in toolkits because there are many models that use it, for example (EPSTEIN, 2006).

In the other side, agent→cell represents the cells one agent controls, targets, watches, or any other objective depending on the context. This representation has a growing use in land use studies, where one agent represents a householder which has to choose what to do with its own territory. As example we can cite (LIM *et al.*, 2002).

Two points are similar in placement relations. They are *binary* relations because, at any given time, two entities are either linked or not. One agent controls or not a cell; one agent is within or without a cell. Placement relations are also *symmetric*; if an agent is within a given cell then the cell contains the agent, if an agent controls a cell then it is controlled by the agent. The same rationale is valid for agent→cell. Therefore we agree with Torrens and Benenson on the working mechanism of these relations, but we propose changing the term geo-referencing to placement because geo-referencing is closer to geometric than to topologic relations, and topology is less restrict than geometry.

2.3.2 Neighbourhood relations

Each relation involving two entities of the same type is a neighbourhood. We have two cases: cell→cell and agent→agent. A neighbourhood between cells represents their spatial proximity. Common representations include the well-known Euclidean neighbourhoods, Moore and von Neumann, which represent proximity through adjacency. But the geographic space by itself is not restricted to Euclidean spaces. Natural features such as mountains and rivers may affect distances, and the human action has significantly contributed to change proximity relations. Neighbourhoods can be created from metrics such as distance, visibility, or accessibility. The result of these metrics can be in the form of weights, suggesting strengths to the connections between cells.

The connection between two agents is often called link, or tie. A link can be based on the time, the emotional intensity, the intimacy (mutual confiding), and the reciprocal services two agents spend with each other (GRANOVETTER, 1973). A recent work shows that spatial proximity remains crucial in effective collaboration, and the technological development mitigates only part of the negative impacts of distance on productivity (KIESLER; CUMMINGS, 2002). Moreover, geography remains crucial even in on-line friendships. A study in a blog network shows that geographic processes draw nearly 70% of on-line friendships (LIBEN-NOWEL *et al.*, 2005), but it does not mean the other 30% results only from non-spatial processes (the Internet, in this case).

A set of interconnected entities, be they agents or cells, form a network. From a network we can extract metrics that describe its structure, providing signatures to compare it with other networks, or even for creating synthetic networks. Geospatial networks can be grouped in classes with strong signatures, distinct from one another and from non-geographic networks (GASTNER; NEWMANN, 2006). Social networks differ from spatial networks (such as road networks) because they are divided into communities, presenting higher clustering levels (NEWMANN; PARK, 2003). Most agent-based simulations use only basic network arrangements in arbitrary ways or as a computational criteria, but "it seems likely that network topology should have some (and as yet unknown) effect on resulting processes and emergent behaviour" (CIOFFI-REVILLA, 2002).

A clear example of using agent→agent relation for building networks within complex

spatial systems is in economic chains models. In these models, we have groups of agents with connections within and between groups. For instance, agents may connect to others according to economic interests to minimise production and transportation costs. Exploring the whole chain and the impact of economic scenarios (for example, governmental investments) over the land use processes is a current challenge of economic models.

Differently from placements, neighbourhoods are not boolean, but *weighted*. The weight of each neighbourhood points out the strength of the connection between the entities. Also, neighbourhoods are not symmetrical, but *directed*. It is possible to have A connected to B with a weight different from B to A. Note that symmetric networks are a subset of directed networks, therefore we can have symmetric neighbourhoods, but not directed placements. Therefore, our approach differs to the proposal of Torrens and Benenson in this case for both characterises: directed and weighted. Table 2.2 summarises the two types of relation of our approach.

TABLE 2.2 - Relations and their differences.

	Neighbourhood		Placement	
	Agents	**Cells**	**Agents**	**Cells**
Flow	Information	Agents, supplies	No flow	No flow
Type	Weighted	Weighted	Binary	Binary
Graph	Directed	Directed	Symmetrical	Symmetrical
Weight	Amount of time, intimacy emotional intensity, reciprocal services, etc.	Adjacency, contiguity, continuity, distance, accessibility, visibility, etc.	—	—

2.4 Requirements for simulating complex spatial systems

Simulating complex spatial systems using ABM needs GIS representations for entities and their relations. The requirements related to the relations are twofold: create and manipulate them. The initial arrangement of each relation can be created within a GIS and loaded from the database to the model. When the relations are previously stored in a database, it is possible to reduce the simulation time whenever we need to replicate the model to verify its convergence, because create some of these relations can be time demanding.

The requirements related to the entities are:

Space. The integration between a toolkit and a GIS should support reading geographic data and their properties, in such a way that the objects of a layer of information have a one-to-one mapping to the spatial objects of the model.

Behaviour. As well as for representing space, agents can be associated to objects stored in geographic databases, with a spatial location and properties. As GIS work with spatial data in an object-oriented way, agents within a given layer have the same properties and the same geometric type.

Time. Besides the scheduler, temporal data that is independent of the behavioural description of the model, usually exogenous events such as building a new road, has to be manipulated directly and transparently by the temporal entity when the data belongs to a geospatial database. The temporal data available in the database has to be automatically loaded before any behaviour take place at any given time step. Temporal changes can be data related to space, agents, or relations. The modeller only has to keep in mind which properties the agents can change, and those which are going to be automatically read from the database, if any.

2.5 Toolkits for agent-based modelling

In this section, we present the most popular toolkits for ABM and their extensions for working with complex spatial systems. We analyse if they support the requirements described in last section. The toolkits are Swarm (MINAR *et al.*, 1997; BOX, 2002), Repast (NORTH *et al.*, 2005; CROOKS, 2007), Cormas (BOUSQUET *et al.*, 1998), OBEUS (TORRENS; BENENSON, 2005), and Netlogo (WILENSKY, 1999). We also study two toolkits developed by Brazilian institutes: GRSP (GRIGOLETTI; COSTA, 2007) and TerraME (CARNEIRO, 2006). Due to space constraints, we do not present an individual description of the toolkits, but a general description of each can be found in their references.

Every toolkit can import some geographic data in order to represent space, but only some of them support agents as geospatial entities. Swarm and NetLogo can load raster data for representing space, in ASCII and PPM (Portable Pix Map)

formats, respectively. Cormas can load data from formats such as MID/MIF and ENV. Repast, OBEUS, and GRSP can represent both space and agents as geospatial entities. Repast uses shapefiles for vector data and ASCII for raster data, OBEUS is a GIS whose objects can change their properties and locations in time, and GRSP accesses PostgreSQL databases. TerraME works with cellular spaces, which are rectangular grids with properties created from raster or vector data stored in TerraLib databases (CAMARA *et al.*, 2000), but it cannot represent agents with geospatial data. None of the analysed toolkits have any control over temporal databases.

In the side of relations, the most common representations available are placement of agents into cells (with an explicit 1:1 agent→cell representation) and neighbourhood of cells, but the toolkits usually do not go further than Euclidean proximities. Exceptions are OBEUS, Repast, and TerraME. OBEUS and Repast can use complex vector operators such as point-in-polygon, buffering and intersection to calculate the proximity between cells. OBEUS uses the leader and follower approach to get agent→agent transitively from other relations. TerraME uses a more general idea of non-proximal spaces. It can access complex neighbourhoods created not only by relations between two objects, but it can also use a third component, for instance a transport network, to calculate proximity. It is called Generalised Proximity Matrix, or GPM (AGUIAR *et al.*, 2003). But, as TerraME does not represent agents as geospatial entities, there is no way to create neighbourhoods for agents.

The lack of complex neighbourhoods in toolkits is due to the difficulty to create these relations from scratch, which can be easily built within a GIS environment. Moreover, it seems that agent→agent relation is commonly left aside because this is the only of the four relations that does not have any direct relation to the geographic space. For example, Repast and Cormas have functions for creating relations between agents through network rules, which are disconnected from the geographic space. Probably that is why Torrens and Benenson resolve this relation using the other relations to make the use of the geographic space explicit. Table 2.3 summarises the capacities of the analysed toolkits.

2.6 Final comments

This work shows that each of the four possible relations for agent-based modelling of complex spatial systems is necessary, and defines a set of requirements to facilitate its use in modelling toolkits. We studied the currently available toolkits and

TABLE 2.3 - Comparison of the toolkits for working with geospatial data.

Toolkit	Space	Time	Agents	Neighbourhood		Placement	
				Agent	Cell	Cell→Agent	Agent→Cell
Cormas	Raster or vector	†	†	Network functions	Euclidean	Manipulate, create	†
GRSP	Vector data	†	Vector data	†	Euclidean	Manipulate, create	†
NetLogo	Raster	†	†	†	Euclidean	Manipulate	†
OBEUS	Vector data	†	Vector data	Transition	Complex operators	Manipulate, create	Manipulate, create
Repast	Raster or vector	†	Vector data	Network functions	Complex operators	Manipulate, create	†
Swarm	Raster	†	†	†	Euclidean	Manipulate	†
TerraME	Cellular data	†	†	†	GPM	†	Manipulate, create

†: Up to the modeller.

verified that they do not support all the requirements presented in this work. The analysed toolkits use geospatial data for representing space, but only some use it for representing agents. There is no complete support for the four relations, and there is no minimal temporal control. The only way to fulfil these requirements is through a GIS integration; otherwise the toolkit would be limited because there is no other way to be always up-to-dated with the novelties in this area.

Currently, we are developing an ABM extension to TerraME to support all the requirements presented in this paper, called TerraME-ABM. TerraME is a toolkit for multiscale modelling of dynamical processes (CARNEIRO, 2006). It is based on Lua (IERUSALIMSCHY, 2003), one extensible and high-level programming language developed by PUC-Rio, in Brazil. We have as hypothesis that the GPM is a foundation for setting up the relations between the entities of an agent-based model for simulating geospatial phenomena.

To corroborate that each of the four relations is necessary, and that the GPM can model these relations, we have as second objective to develop models using the three relations that involve agents. They are:

cell→agent: We have developed a theoretical model that simulates a competition for space, where agents fight for the cells they belong through a non-cooperative game. We propose a definition of mobility, and study how

it can affect the results of a given model. This work was submitted to the Journal of Artificial Societies and Social Simulation. The objective here is to build a spatial database with the initial arrangement of the model. With it, the model will only load the arrangement, define the behaviour of the agents, and then perform the games.

agent→cell: The centre-north region of the Rondônia state, in Brazil, has its occupation history associated to colonisation projects, induced migratory flows, the BR-364 railway construction, and the establishment of development poles (BECKER, 1997). The objective is to study the deforestation trajectory in this region from individual decisions, modelling agents according to their farm size: small and large (ESCADA *et al.*, 2005). There is an early work that uses cellular automata (CARNEIRO *et al.*, 2004). The objective now is to model agents which decide what to do with their own sets of cells.

agent→agent: The açaí occurs naturally in the Amazonian floodplain forest, providing both fruit and the so-called heart of the palm, or palmito (BRONDIZIO; SIQUEIRA, 1997; BRONDIZIO, 2008). The objective is to study the açaí economic chain, in Pará, Brazil, exploring questions such as its development and sustainability. This model will be more complex than the other two, once it involves agents in different scales, and it will not be limited to only the agent→agent relation.

3 AN ARCHITECTURE FOR AGENT-BASED MODELLING AND SIMULATION OF GEOSPATIAL PHENOMENA[1]

Abstract

This work presents a general architecture for building and simulating agent-based models that use real-world geospatial data, taking into account all the ways geospatial data can feed these models. We focus on how data can be used to create an initial arrangement for the model, as if it was a static representation. We have as hypothesis that the Generalised Proximity Matrix (GPM) is a foundation for setting up the relations between the entities of an agent-based model for simulating geospatial phenomena. The architecture has been implemented in the TerraME framework, which is capable of exchanging data with TerraLib databases. Both softwares are freely available in the Internet.

3.1 Introduction

Representing the micro-behaviour in order to get a macro-pattern is one of the key challenges of agent-based modelling (ABM). Since its beginning, real world evidences have guided the development of both agents and the environment with which they interact. This leaded to the development of artificial worlds to understand some underlying principles of human behaviour, such as wealth distribution and evolution of cooperation (EPSTEIN; AXTELL, 1996; AXELROD, 1997b).

On the other hand, many models have to be less general and more precise (COSTANZA *et al.*, 1993), mainly when we study decision making processes related to the geographic space, as in the case of land use and land cover change. In these models, the agents have to be grounded to a representation of the world, shifting from theoretical abstractions to case-based models that focus on populations within specific localities. With it, agent based modelling has facing the challenge of us-

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[1] This chapter is an extended and slightly revised version of the paper co-authored with A. M. V. Monteiro, G. Camara, and T. G. S. Carneiro, *"An Architecture for Agent-based Modelling and Simulation of Geospatial Phenomena"*, **Proceedings of the 6th European Social Simulation Association Conference (ESSA'09)**, 2009.

ing not only real world evidences but data (BOERO; SQUAZZONI, 2005), especially geospatial data (GILBERT, 2008), to feed models. Brown and others have argued that "incorporating real spatial heterogeneity into agent-based models will improve our ability to draw conclusions about the behaviour of complex systems in realistic environments" (BROWN *et al.*, 2005). This leads to a need towards integrating toolkits for agent-based modelling and Geographic Information Systems (GIS), since the formers need the spatial heterogeneity and the latters are not well suited for representing and studying behaviour (GOODCHILD, 2005; CASTLE; CROOKS, 2006).

However, most of the available toolkits were initially conceived to work with landscapes that do not consider the environment in which we move and interact. They rely on non-grounded agents, in the sense that they do not represent entities which can sense and act in a representation of the real world. It makes implementing and simulating models based on the geographic space a difficult task, once the modeller has to program all the necessary geospatial data input, output, and control.

To accomplish this challenge, toolkits have to carry out the support for working with geospatial data. However, the demands and formalisms for creating agent-based models at this complexity level are not well established in the literature. There is a couple of works proposing general requirements and analysing toolkits (GASSER, 2000; MARIETTO *et al.*, 2003; NIKOLAI; MADEY, 2009; RAILSBACK *et al.*, 2006; TOBIAS; HOFMANN, 2004), but only a few concern integration with GIS (CASTLE; CROOKS, 2006; ANDRADE *et al.*, 2008). Probably because of this, newest toolkits, as well as spatial extensions of older toolkits that were developed before realising the need of working with geospatial data, have been created according to the class of problems they are trying to support, instead of following a conceptual and more general approach. For example, common approaches represent landscapes where artificial agents move and interact using geospatial data (BARROS, 2004; BOX, 2002; CROOKS, 2007), or use data to feed agents that move in artificial environments (CROOKS, 2007), but they do not use geospatial data to connect them.

The objective of this work is to present an architecture to allow implementing and simulating agent-based models, taking into account all the ways geospatial data can feed these models. We focus on how data can be used to create an initial arrangement for the model, as if it was a static representation. Our working hypotheses is that the idea of Generalised Proximity Matrix, or GPM, is a foundation for setting up relations between the entities of agent-based models that simulate geospatial

phenomena. This work is written as follows. Section 3.2 presents some concepts and related works. In Section 3.3, we describe the idea of GPM, and how it can be used to set up the relations involving the entities of agent-based models. The architecture is described in Section 3.4, and finally Section 3.6 presents the concluding remarks and future works.

3.2 Related works

Brown and others present four types of relations where geographic data and agent-based models can interact (BROWN *et al.*, 2005). *Identity* relations points out that an agent can be associated with one or more spatial features. Agents have the ability to take actions that affect spatial features and/or their attributes, which is described as a *causal* relation. Changes are essentially time-sensitive, generating a *temporal* relation. Finally, *topological* relations are queries about the relations among spatial features and spatial associations between features such as distance or visibility.

Torrens and Benenson work with identity and topological relations (TORRENS; BENENSON, 2005). They argue that the two basic types in models that simulate geographic processes are *cells* and *agents*. They define *relation* as a mapping that describes the connection between two entities[2]. As there are two classes of entities, then we have four types of relations: cell→cell, agent→cell, cell→agent, and agent→agent. The authors use a leader and follower approach to define spatial relations. Leaders are responsible for managing relations, while followers are a passive objects. The aim is to keep consistency in the relations, avoiding conflicts between the entities. Therefore we have cell→cell as a spatial neighbourhood, a static relation (two followers). Agent→cell represents the cells one agent controls, while cell→agent describes the agents which belong to a certain spatial location. Finally, agent→agent represents the relations between agents. According to the authors, agent→agent can only be generated by transitivity over the other relations, once this relation has two leaders and it is impossible to define who owns the relation in order to guarantee consistency.

Andrade and others grouped these relations in placements and neighbourhoods (ANDRADE *et al.*, 2008). Given the two entities, cells and agents, placements connect entities from different types (agent→cell and cell→agent), while neighbourhoods connect entities of the same type (cell→cell and agent→agent). The authors argue

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[2]In this work, we adopt this definition for the term *relation*.

that the four relations are necessary and can be fed by geospatial data, in spite of the fact that the toolkits currently available do not support all four types of relation. However, the authors do not present any conceptual framework to work with these relations.

In this work, we consider that, instead of agents having to query a geographic database whenever they need an answer about a specific spatial structure, the representation of space within the model is already filled with this data. It represents a suitability map, where the neighbourhoods between cells represents criteria such as visibility or possible movements. We assume the modeller previously knows the queries the agents may perform along the simulation. It can be considered a limitation of this approach, but it is a way to see the problem of using geospatial data, with the advantage of separating GIS functions from the simulation. The idea is that both applications can work harmonically but separately, sharing only geospatial data.

Filling the whole space with the results of the queries commonly requires more time than performing a couple simulations. However, once the data is already in the database, the simulation runs faster because there is no need to maintain a connection to a geographic database to perform such queries. It can also be considered an advantage, once replication is a common procedure for studying the overall behaviour of the model. To accomplish that, we need a glue to define the relations between the entities of agent-based models. We propose the idea of GPM as a way to establish these relations.

3.3 GPM as basis for setting up relations

Based on the fact that Euclidean spaces are not satisfactory to describe the underlying complexity of the relations of the geographic space, Aguiar and others present the idea of Generalised Proximity Matrix, or GPM (AGUIAR *et al.*, 2003). The GPM combines data from Euclidean spaces and from topological relations embedded in these spaces as a way to measure spatial relations between geographic objects. The topological relations warp the Euclidean space, reducing distances. One straightforward example is the case of transport networks.

There are two strategies for computing distances based on topological relations using GPM. They are called *open* and *closed* networks. Open networks can use any location in the edges as entry points. In closed networks, the nodes are the only entry points
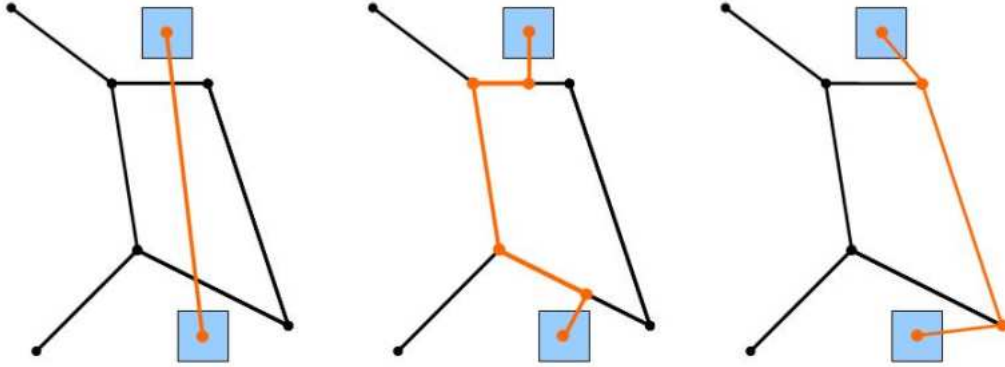
FIGURE 3.1 - Strategies for creating relations using GPM. From left to right: (a) Euclidean spaces, (b) open networks, where it is possible to enter the network in any point, and (c) closed networks, where it is possible to enter the network only in some predefined points.

to the network. Figure 3.1 shows two squared objects and the different ways to calculate their proximity, using Euclidean spaces and the two types of networks.

GPM establishes relations between sets of geospatial entities based on geometrical relations. The result is a graph connecting these entities, which can feed each of the four types of relations discussed in the last section. Metrics of distance using Euclidean spaces or networks can be used to create neighbourhoods because proximity is the base for these relations when we use the geographic space. Placements are different from neighbourhoods because the objects must have some overlay. Generally, one object has a fine and the other has a large resolution. In the case of agent→cell, agents have at least a resolution as large as the resolution of the cells, and the opposite happens in the case of cell→agent. Therefore we can treat placements as multiscale relations. GPM has some strategies to calculate the relations between objects represented as rectangular cells (MOREIRA *et al.*, 2008), which can be extended to work with polygonal objects.

Examples of using GPM to feed agent-based models are shown in Table 3.1. In the cell→cell case, upper neighbourhoods are created from a Euclidean space, while the lower ones use the network to calculate the nearest cells. Agent→cell and cell→agent are overlays between squared cells and agents represented by polygons and points, respectively. Agent→agent is a relation between pairs of points measured from a specific vision radius.

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TABLE 3.1 - GPM as a graph. Placements are bipartite graphs built from relations in Euclidean spaces, while neighbourhoods can be generated from relations in non-Euclidean spaces, represented by graphs that can have properties such as small worlds or scale-free networks.

From \ To	Cell	Agent
Cell		
Agent		
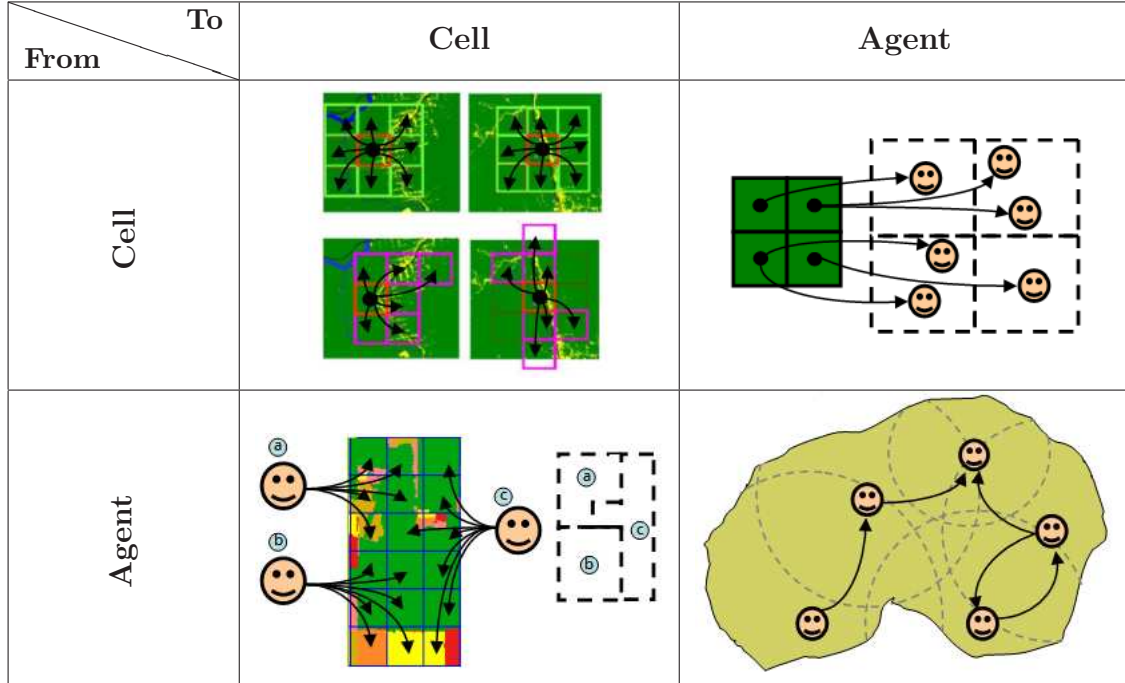
3.4 The agent-based modelling architecture

The main components of the proposed agent-based modelling architecture are:

Agent and Cell. The basic geospatial entities, each one with its own set of properties and relations. Cells are passive entities, while agents are flexible and autonomous.

Society and CellularSpace. Sets of agents and cells, respectively. Agents within the same Society have the same set of properties and general behaviour. CellularSpaces represent cells with the same set of properties and spatial resolution.

Group and Trajectory. Subsets of Societies and CellularSpaces, respectively. They can be ordered whenever it is possible to have a function that compares pairs of objects. Both can be used to create placements and neighbourhoods from non-spatial attributes, and can be traversed as if they belong to the same type of the objects from which they were extracted.

Geospatial data. Real world as well as artificial geospatial data can be used to feed entities and relations of agent-based models.
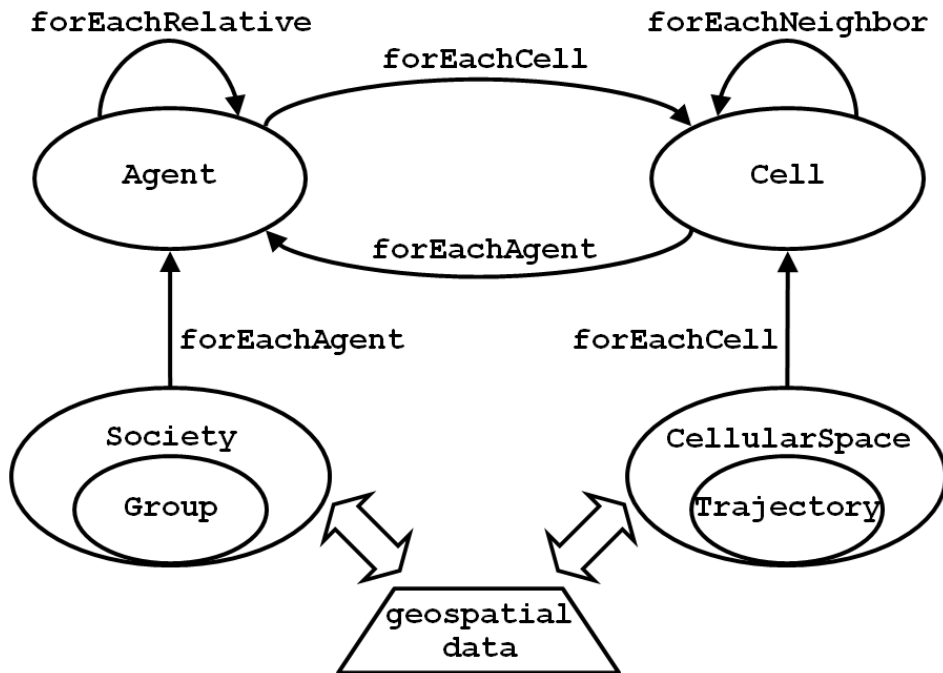
FIGURE 3.2 - Schema with the main components of the architecture and their relations.

Figure 3.2 shows the main components of the architecture, with arrows representing a relation between two given components, each one being 1:n. In order to transverse these relations, we propose having second order functions that can be used for different relations. For example, `forEachAgent` can traverse the agents within a Society as well as the agents that belong to a given cell.

3.5 Implementation

The architecture has been implemented as part of the TerraME framework. TerraME is a software environment for spatially explicit dynamical modelling (CARNEIRO, 2006). A spatial dynamic model is a model whose locations are independent variables. The outcomes of these models are maps that depict the spatial distribution of a given phenomenon.

The internal structure of TerraME is composed by a hierarchy of layers, where lower layers provide basic services over which upper layers build new services. The lowest layer is TerraLib, a C++ open source GIS library (CAMARA *et al.*, 2008). It provides typical spatial data management and analysis services, and procedures for temporal data handling. TerraLib stores data in layers of information, each one representing a set of geographic objects with the same attributes. It also stores the results of GPM, pointing out the connections between objects and the layer(s) they belong to.

The core for the simulation engine and data interface comes directly above TerraLib. From that, TerraME uses a Lua interface to use the concepts designed in the TerraME framework as a Lua extension, with which final users can write their models. Lua is an extensible and high-level scripting language (IERUSALIMSCHY, 2003). It has a great acceptance on the games development community due to its simple way to manipulate data and easy interface with other languages.

3.5.1 Data interface

Given TerraLib definitions, the entities of the proposed architecture have the following characteristics related to data interface:

- Society and CellularSpace have properties to create a connection to a database and to identify in which layer of the database the data is going to be read;

- Agents and cells have unique identifiers to name which objects stored in a layer of a geographic database they represent;

- Relations are also stored in the database, using tables with pairs of elements connecting objects from one or two layers.

Before the modeller runs the simulation, she has to prepare the database with the geospatial data and their relations, generating GPMs and selecting all the necessary data from different layers. The simulation can change properties and relations, and the results can be stored in the database for further analysis, possibly using TerraView GIS[3] or the R statistical software (ANDRADE *et al.*, 2005). The interface with these programs is transparent because they can access TerraLib databases directly, without having to export data to a different format.

An example of source code for loading a Society, a CellularSpace, and their relations from a given database is shown in Figure 3.3. As data is stored in a database, the modeller only needs to point out from where the data is going to be loaded. The difference between creating a Society and a CellularSpace is the presence of a function to describe the behaviour of the agents. This function, in this case called `createAgent`, receives a data structure containing the properties of a single geospatial entity and returns an agent built from these properties. The modeller has the

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[3]http://www.dpi.inpe.br/terraview/

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option to select which attributes are going to be read from the database (in the example of `myCellularSpace`, it will read `cover` and `soil_quality`), otherwise it will read all the available attributes (as in the case of `mySociety`). TerraME automatically loads the data according to their specific types in the database table, avoiding any type matching between the database and the source code.

```
createAgent = function(data)
    a = Agent(...)
    -- ...
    return a
end

mySociety = Society {
    database = "d:\\database.mdb",
    layer = "farmers",
    instance = createAgent
}

myCellularSpace = CellularSpace {
    database = "d:\\database.mdb",
    layer = "cells",
    select = {"cover", "soil_quality"}
}

myEnvironment = Environment {
    mySociety,
    myCellularSpace
}

mySociety:loadNeighbourhood("nearest_farmers")
myCellularSpace:loadNeighborhood("transition_by_roads")
myEnvironment:loadPlacement("farms_to_cells")
```

FIGURE 3.3 - Loading entities and their relations from a geographic database.

Once both objects are created, neighbourhood relations can be loaded directly from them, as also shown in Figure 3.3. A call to `loadNeighbourhood` connects the entities *within* a Society or a CellularSpace, while `loadPlacement` sets up the relations *between* a Society and a CellularSpace. In the case of placement relations, as they involve objects from different types, we need a third object, of type *Environment*,

to encapsulate both in a single object and then connect them. When loading the relations, TerraME automatically identifies the layers according to the relation stored in the database, `farms_to_cells` in the Figure. Then it searches the Environment for the objects that match the layer names and connects the pairs of objects. In this sense, it is possible to have more than one Society in an Environment and TerraME still finds out and links the correct entities.

3.5.2 Direct and indirect relations

The GeoDBMS contains geospatial data to feed the relations involving agents and/or cells. These relations are stored directly within the entity that contains it, but there may exist relations that are not stored directly. For example, Torrens and Benenson consider that relations between agents can only be solved indirectly as a transition over the other relations (TORRENS; BENENSON, 2005). With this in mind, TerraME is more flexible because it is possible to choose between direct and indirect relations. Direct relations are stored by a data structure within the entity and can be fed by geospatial data, while indirect relations are queries over direct or indirect relations. To accomplish that, each `forEach` of Figure 3.2 uses a `get` function that returns the data structure directly stored or calculate the relation on-the-fly. For example, Figure 3.4 shows a simplified version of `forEachAgent` that calls `getNeighborhood`, a function that by default returns a direct relation.

```
function forEachAgent(object, func)
    n = object:getNeighborhood()
    for i = 1, n:size() do
        if not func(object, n[i]) then return false end
    end
    return true
end)
```

FIGURE 3.4 - A simplified version of `forEachAgent`. It gets the neighbourhood structure of the object and then transverse it. If some function call returns false along the processing, `forEachAgent` stops and does not process the other agents.

Whenever an indirect relation is necessary, the modeller only needs to overwrite the `get` function (in this case, `getNeighborhood`) by a query over the other relations. The two indirect neighbourhoods between agents proposed by Torrens and Benenson

```
agent.getNeighborhood = function(self)
    n = Neighbourhood()
    forEachCell(self, function(agent, cell)
        forEachAgent(cell, function(other_agent)
            if agent ~= other_agent then
                n:add(other_agent)
            end
        end)
    end)
    return n
end
```

FIGURE 3.5 - Neighbourhood of agents within the same cell. The agent needs to avoid itself as it belongs to the same cell of its neighbours. Commonly, agents belong to a single cell, turning `forEachCell` unnecessary. But it becomes necessary when the agent represents an institution that can belong to more than one spatial location at each time instant.

```
agent.getNeighborhood = function(self)
    n = Neighbourhood()
    forEachCell(self, function(agent, cell)
        forEachNeighbor(cell, function(neighbor)
            forEachAgent(neighbor, function(other_agent)
                n:add(other_agent)
            end)
        end)
    end)
    return n
end
```

FIGURE 3.6 - Neighbourhood of agents belonging to adjacent cells. The idea is to transverse all neighbours of the current cell, retrieving the agents within these cells.

can be implemented as shown in Figures 3.5 and 3.6. With it, one can simply use `forEachNeighborhood` as if it was a direct relation. Therefore, it becomes transparent for the modeller whether it is directly stored or indirectly calculated: the way of retrieving the relation is the same for both cases.

3.6 Concluding remarks

Simulating geospatial phenomena demands reliable toolkits that allow the user to focus on describing the model, rather than worrying about data structures and

simulation control. When we add a spatial component to these systems, with the objective to represent entities and their relations with real-world geographic data, modelling becomes an even harder task, commonly requiring strong computational abilities to build new up models. To make this task easier, we need simple but robust toolkits that generalise the representations of geographic data for creating agent-based models.

In this work, we presented an architecture where it is possible to use geospatial data stored in a database to create agents, a representation of the geographical space, and relations involving them. The approach does not focus on a specific type of application, such as urban or ecological applications. Instead, the objective is to have a general architecture for models where topological relations involving agents and cells matters, allowing the modeller to create relations directly from data or indirectly through other direct or indirect relations. With this, the approach presented in this work is more general than the others available in the literature.

Future directions of this work are related to a temporal representation of geospatial data. The requirements for the entities and relations have to be better understood, and new requirements will show up. Another future work involves using this architecture to develop land use and land cover change models in the Brazilian Amazonia.

4 GAMES ON CELLULAR SPACES: HOW MOBILITY AFFECTS EQUILIBRIUM[1]

Abstract

In this work we propose a new model for spatial games. We present a definition of mobility in terms of the satisfaction an agent has with its spatial location. Agents compete for space through a non-cooperative game by using mixed strategies. We are particularly interested in studying the relation between Nash equilibrium and the winner strategy of a given model with mobility, and how the mobility can affect the results. The experiments show that mobility is an important variable concerning spatial games. When we change parameters that affect mobility, it may lead to the success of strategies away from Nash equilibrium.

4.1 Introduction

Over the last years, the role of spatial self-structuring in the study of games has drawn a lot of attention. Particularly, games with spatially explicit factors have proven to be useful for modelling biological and economic environments (NOWAK; SIGMUND, 2000). The purpose of such games is to assess the effects that spatial structures have on adaptation strategies of agents, mainly in the study of the evolution of altruistic behaviour. Adding a spatial component to the models (usually in the form of grids) often displays different features from models with well-mixed populations. For example, the evolution of interspecific mutualism cannot be explained by an unstructured population through the iterated continuous Prisoner's Dilemma (SCHEURING, 2005).

Most of the games on grids use a straightforward extension of non-spatial games, with each cell containing a single agent that interacts with its neighbourhood. If the agents interact only with fixed neighbours, there will be no need to recognise and

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remember the opponents (NOWAK *et al.*, 1995). This arrangement allows the same game to be repeated, and the results influence how the agents populate the cellular space. As an example, Nowak and May study the spatial Prisoner's Dilemma, in which a cell is given to the strategy with highest payoff in the neighbourhood, if it is greater than the current payoff (NOWAK; MAY, 1992).

A spatial arrangement may lead to different results because it changes the way with which agents interact, from well-mixed confronts to local competitions. But there is a second characteristic of space that may affect the development of spatial models: the *mobility*. Agents can move in space, changing their spatial locations and making the local relations dynamical.

The literature of spatial models presents two definitions of mobility. First, analytical models using replicator-diffusion equations often call mobility the spread of a strategy in space (FERRIÈRE; MICHOD, 2000). The second definition works with the mobility of an individual agent, which can move to an empty neighbour cell. Examples of such models can be found in studies of cooperation (EPSTEIN, 1997; GALLIARD *et al.*, 2003), linguistics (KOSMIDIS *et al.*, 2005), and evolution of cancer (MANSURY *et al.*, 2006).

In all works that involve numerical simulation, the mobility of an agent is an automated action, which always depends on a social opportunity as, for instance, an old neighbour having left an empty cell. Moreover, that happens even if the current spatial location is favourable to the agent. However, instead of moving to the neighbour cell as soon as possible, a "smart" agent would ask: why should I move if I am getting good results? That is a common strategy adopted in spatial models which do not use games, such as in (ZHANG, 2004) and (BELTRAN *et al.*, 2006). In these works, the agent compares the current satisfaction with the satisfaction in the new spatial location, moving only when its satisfaction increases with the change.

In this work, we explore the question of mobility within the context of non-cooperative games, more precisely the chicken game. We propose a new spatial model by defining mobility in terms of the satisfaction of an agent with its current spatial location. Agents use mixed strategies to compete, and decide to move according to their payoffs. An agent does not depend on a social opportunity for leaving its spatial location: it can move to a new spatial location even if it is already populated with other agents. Agents have complete freedom to move; there is no cost or

constraint to do so. In this framework, instead of competing with their neighbours, the agents compete with each other within the same cell.

Contrary to what happens in the literature of spatial games, in this work we have a clear separation between the concepts of cell and agent. There are many real world situations where it is useful to consider agents moving from one cell to another. Such cases are common in geographical space, where a cell is usually a container that has properties which might be different from its neighbours. Take the case of land use change models. Land use change cells differ not only in their locations, but also in properties such as topography, water availability, and temperature. In these and similar cases, a cell can contain a small community of agents. Agents interact with their community (inside the cell) and if dissatisfied, they will try to find a more friendly community elsewhere in space. Combining interaction within cells and movement between cells allows new insights when modelling games in space.

We are particularly interested in studying the relation between the Nash equilibrium and the winner strategy of a given spatial model with mobility, and how the definition of mobility can affect the results. In a nutshell, we try to answer the following questions: is Nash equilibrium the best strategy in a competition for space where the agents can move according to the results of the games? If not, in which cases, and how can it help to define the best strategy? Also, how does the proposed definition of mobility possibly affect the results?

4.2   Non-cooperative games

A non-cooperative game is an n-person game where the actions of each player are independent, without any collaboration or communication with the other players. In an n-person game we have, for each player:

a) a finite set of pure strategies (actions);

b) a payoff function, that maps all n-tuples with the individual pure strategies to real numbers.

One mixed strategy is a collection of non-negative numbers adding up to 1, corresponding to probabilities of using each of the pure strategies. The mixed strategy defines the *tendencies* of a player. Each time it plays, it will choose randomly one of its pure strategies, based on the probabilities defined by the mixed strategy.

For example, let us take the chicken game. Two players have the choice to escalate (E) or not to escalate ($\tilde{E}$) a brawl. If none of them escalates, nothing happens. If only one escalates, the other player runs away, and the winner receives 1 from the coward player. But, if both decide to escalate, each player pays 10 due to medical care. This game is said to be symmetric, because both players employ the same pure strategies and payoffs, as shown in Table 4.1. Given that this game has only two pure strategies, we denote by s_x, $0 \leq x \leq 1$, the mixed strategy of escalating with probability x.

TABLE 4.1 - Chicken game payoffs, in pairs (A, B).

	B escalates	B does not escalate
A escalates	$(-10, -10)$	$(+1, -1)$
A does not escalate	$(-1, +1)$	$(0, 0)$

Nash proved that, given any non-cooperative game of n players, there is always an equilibrium point. This point is a set of mixed strategies for each player that, if a player individually changes its mixed strategy, the best result it may get will be the same as in the equilibrium (NASH, 1951). No player has incentive to deviate one-sidedly from its strategy as long as the other players remain in the equilibrium. This is known as the Nash equilibrium.

For example, let A and B be two players of the chicken game, with strategies s_a and s_b, respectively. The expected payoff of player A is $-10s_a s_b + s_a - s_b$. If A knew exactly the value of s_b, it would be possible to calculate the best action for it. If s_b is greater than 10%, the best choice for A is never to shoot ($s_a = 0$), implying in a payoff of $-s_b$. If s_b is less than 10%, A should always shoot ($s_a = 1$), because its payoff would be $1 - 11s_b$. But, if s_b is exactly 10%, all strategies for A lead to the same payoff (-0.1). Therefore, if s_a is also fixed at 10%, no other strategy could augment its payoff against A by changing its own mixed strategy. Applying the same reasoning for B, we arrive to the conclusion that when both players follow $s_{0.1}$ the game is at Nash equilibrium.

But this idea of equilibrium may cause controversy in some games. Most game theorists agree on $s_{0.1}$ as the rational solution for this game, but the argument is somewhat tenuous (SIGMUND, 1993). In the chicken game, although deviating from

the equilibrium does not increase the utility of the player, it does not decrease as well, as long as the other player stays in the equilibrium. Therefore, it is not a *strict* equilibrium.

A clear explanation can be found when it is played not only by two players, but within a population. Maynard Smith viewed this game in a population-dynamical setting (MAYNARD SMITH, 1982). In his model, a large number of players meet randomly in contests where they have to decide whether to escalate or not. If the estimated overall probability is greater than 10%, it is better not to escalate. If it is less than 10%, it is better to escalate. But if it is exactly 10%, then there is no better strategy than $s_{0.1}$. In this sense, self-regulation leads to $s_{0.1}$ – self-regulation not between two players, but within a population. Nash has also proposed a similar interpretation for equilibrium points, the mass-action (NASH, 1950), which was forgotten for decades in his unpublished thesis. In this work, we will study how this non-strict equilibrium behaves in a spatial context with the agents' mobility based on the results of the games.

4.3 Related works

Recently, new models with different neighbourhood topologies have been proposed to study games in space. The proposals include variations of grids, graphs, and some in between structures (BIELY *et al.*, 2007; CASSAR, 2007; DURÁN; MULET, 2005; SOARES; MARTINEZ, 2006; VAINSTEIN; ARENZON, 2001; VUKOV; SZABÓ, 2005; WU *et al.*, 2006). Ohtsuki and Nowak have shown that, in the limit of weak selection, models with different topologies can be described only by changing the payoff matrix (OHTSUKI; NOWAK, 2006). Although these models have different topologies, agents have the same characteristics: they are fixed in the spatial partitions, have only a pure strategy, play with neighbours, and may spread their strategies to the neighbourhood according to the result of the games.

Commonly, the result of the game is only used to define a new spatial arrangement on the next generation of the model, but it does not have any future effect in the simulation: once the competition ends and the best strategies replace the worse ones, the simulation restarts again as if the games were not played. Feldman and Nagel propose a model where cells are not updated by a new strategy at the end of the turn (FELDMAN; NAGEL, 1993). Instead, each cell must pay a fee each turn to stay in the model, and a neighbour strategy will replace it only if its savings end. Epstein

studies the evolution of cooperation in a model where agents may be removed from the simulation if they reach zero or less wealth, since the payoff matrix has negative entries (EPSTEIN, 1997).

Beltran and others propose a model where agents move in a lattice trying to minimise their dissatisfaction (BELTRAN *et al.*, 2006). The satisfaction is a function of the difference between the real distances it keeps from the other agents and the ideal distances it wants to keep from them. (ZHANG, 2004) studies a segregation model, where agents may exchange their spatial locations according to a definition of satisfiability. In both models, only one agent can be within a cell in each time step.

4.4 The proposed model

The model takes place in a cellular space. A cellular space is an environment with cells connected by neighbourhood relations. The simplest example of a cellular space is a grid, with square cells having four touching neighbours. The objective of using the term cellular space instead of graph is to distinguish the meaning of a neighbourhood: instead of connecting individuals, as in typical spatial games, a cellular space simply connects cells, defining a spatial proximity relation.

The cellular space is always populated with individuals called *agents*. Each agent belongs to a single cell, which has enough space for it to live. Initially, each cell contains a set of agents, which have to compete for space through a non-cooperative game. Whenever an agent is playing a non-cooperative game, we call it *player*, but one agent has other characteristics besides those of players in the sense of a non-cooperative game (pure and mixed strategies), as it will be seen below.

The basic assumption of our model is that when an agent arrives at a cell (as well as in the beginning of the model), it is satisfied with its cell, and thus *no agent will move unless it is dissatisfied*. Two agents within the same cell may play a non-cooperative game competing for it, and the result of each game affects directly their individual satisfactions with the current cell. This is the only memory an agent has, being called *local satisfaction*. It starts with a positive value when an agent arrives at a cell. Whenever it reaches zero or less, the agent randomly picks a neighbour cell and moves to it, looking for a better cell to compete for. Given that, the movement of agents in a cellular space can be characterised as a random walk.

Each agent also has a *global satisfaction*, starting with a positive value significantly

greater than the local satisfaction. As the local satisfaction, it is affected by the payoffs of the games. All agents have the same global satisfaction at the beginning of the model. An agent that got dissatisfied many times and its global satisfaction reaches zero or less leaves the model. In the models we propose in this work, the global satisfaction does not affect the agents's behaviour.

To create a metric for measuring satisfaction, we say the satisfaction of an agent is measured by its *capital*. Local satisfaction represents the limit of capital one can dispend for a cell. Global satisfaction corresponds to the initial capital assigned to an agent, thus it leaves the model when its money ends. As global and local satisfaction and game payoffs are strongly related to each other, throughout this text we use the symbol $ to represent any value related to them.

Agents differ in one characteristic: the mixed strategy. We divide the agents in groups of equal size. Two agents within a group share the mixed strategy, but they cannot communicate nor identify each other. Although the agents compete individually, they represent a strategy that is going to be studied in a spatial context. To represent the fittest agents along the model execution, the agent with higher global satisfaction in a cell will be its owner, because, at least at first sight, it has more chance of surviving than any other in that cell. If two agents have the same amount of money in a given cell, the agent that stands there for a longer time is its owner. In the first turn of the model, we divide the owners equally among each group.

The model has a finite number of turns, each one with two steps. The first step sets up and carries out the games. Supposing the game has two players, we randomly choose pairs of agents in each cell, and then carry out the game with each pair. Cells with an odd number of agents have one random idle agent, and no agent will play more than once in each turn. The same logic can be applied to games with more than two players.

The second step defines the dynamical part of the model. Once each agent already knows its own payoff, it updates its local and global satisfactions with the payoff. Then, it checks if any satisfaction has reached zero or less to perform a movement or leave the model. The model runs until it reaches a stable state, which happens when there is at most one agent in each cell, or when their satisfactions stop to decrease. Whenever the model arrives at one of these situations, we say that it is at equilibrium.

There are two differences between the general model proposed here and the ones proposed in the literature. First, and most importantly, the model separates agents from space. The agents compete for space but they are not equivalent to the space itself. Neighbourhood relations point to where an agent can go, trying to find a better cell to fight for. Second, an agent plays with a random opponent inside the same cell and, as agents can move, the chance of two agents meet more than once can be very small. Given that, we only allow mixed strategies, and no meta-strategies such as Tit-For-Tat (AXELROD, 1980) or Pavlov (NOWAK; SIGMUND, 1993), because this model is not a repeated game.

A model of games on cellular spaces can be formalised as a 8-tuple:

$$M = (C, n, S, p, A, k, g, l), \text{ where}$$

- C is the cellular space in which the games take place;

- $n \in \mathbb{N}$ is the number of players involved in the non-cooperative game;

- S is the set of actions (pure strategies) each player can take;

- $p : X \to \mathbb{R}^n, X = \{(x_1, \ldots, x_n) | x_i \in S\}$, is the payoff function;

- A is the set of groups of agents, each one represented by a mixed strategy;

- $k \in \mathbb{N}$ is the initial number of agents of each group;

- $g \in \mathbb{R}$ is the global satisfaction;

- $l \in \mathbb{R}$ is the local satisfaction.

Given the characteristics of the model, it is necessary to note that:

- $|A|$, the number of groups, is not necessarily related to n;

- All the agents play the same game, obeying the rule that each game requires n players to be executed;

- In order to keep the probability of a group to survive depending only on its mixed strategy, it is necessary to have a symmetric game;

- An agent using a mixed strategy commits itself to a randomisation device. Each time the agent plays, it chooses a pure strategy based on the probabilities specified by the mixed strategy.

As an example of model, the following one denotes the traditional chicken game:

$$M_c = (C_1, 2, \{E, \tilde{E}\}, u, \{s_a, s_b\}, 1, \infty, \infty).$$

The cellular space C_1 has a single cell containing two agents, each one coming from a different group, one from s_a and the other from s_b. They have the same set of possible actions, "escalate" (E) and "not escalate" ($\tilde{E}$), and the same payoff matrix u (shown in Table 4.1), but they do not have to follow the same mixed strategy. Both agents always stay in the only cell and never leave the model.

In this work, we are particularly interested in applying the chicken game within this general model. The expected payoff of this game is almost always negative, only in the case where both players never escalate the expected payoff is 0. Therefore this game fits in with the requirement of reducing their satisfaction to make them move. We use the cellular space named C_{20}, a squared grid with 20×20 cells, such that the possible movements of an individual are at most to four neighbours (up, down, left and right). Cells on the edges have three alternatives for movement, and cells on the corner have only two.

4.5   A first experiment

As first experiment, we divide the agents equally in the following three groups:

a) Always use the pure strategy seemingly more profitable, escalate, because it is the only way to earn something, and the opponent will have a payoff at most as bad as its;

b) Choose randomly a pure strategy in each game (escalate with chance of 50%);

c) Follow Nash equilibrium, escalating with chance equal to 10%.

Initially, there are three agents of each strategy competing in each cell, summing-up nine agents by cell. Each agent starts the model with \$200 and, inferring that losing

TABLE 4.2 - Example of turn in a given cell.

Agent	Strat-egy	Beginning		Games			End of the turn		
		Global	Local	Confront	Action	Payoff	Action	Global	Local
a	$s_{0.5}$	17	4	Idle	Idle	$+0$	Stay	17	4
b	$s_{0.5}$	15	1	$b \times d$	$\tilde{E}$	-1	Move	14	$0 \to 20$
c	$s_{0.1}$	21	15	$c \times e$	E	-10	Stay	11	5
d	$s_{1.0}$	37	12	$b \times d$	E	$+1$	Stay	38	13
e	$s_{0.5}$	10	5	$c \times e$	E	-10	Dye	$-$	$-$

$10 twice without earning any money is enough to turn an agent dissatisfied, we chose $20 to be the local satisfaction. Therefore we have:

$$M_0 = (C_{20}, 2, \{E, \tilde{E}\}, u, \{s_{0.1}, s_{0.5}, s_{1.0}\}, 1200, 200, 20),$$

where $s_{0.1}$ follows Nash equilibrium, $s_{0.5}$ plays by using a coin toss, and $s_{1.0}$ always escalates.

To exemplify the way that this model executes, suppose a turn of a given cell with five agents (a, b, c, d, and e), as shown in Table 4.2. The model randomly selects the pairs (b, d) and (c, e) to play chicken, leaving a idle because the cell contains an odd number of agents. Let us analyse the two games separately. In the first game, b follows $s_{0.5}$ and decides not to escalate, while d decides to escalate due to its strategy $s_{1.0}$. With this, b loses $1 and d gets $1. After the game, b has $14 and its satisfaction is 0, while d has $38 and its satisfaction is 13. Consequently, b reaches the moving threshold, leaves the cell, and updates its local satisfaction to 20 again. Finally, d decides to stand still.

In the second game, c against e, both decide to escalate. This causes a lose of $10 for each agent. After the game, c has $11 and its local satisfaction is 5, while e has $0 and its local satisfaction is -5. Consequently, c decides to stand still, while e leaves the game because its fitness has reached a value less than zero.

As we can see in the example above, this model is stochastic, and we are interested in the convergence more than in showing numerical results. Clearly, M_0 always converges to the state where there is at most one agent in each cell, but when we say convergence, we aim at verifying whether the results obtained by each strategy in the simulations are similar.

We run the model 50 times to verify convergence and, although there are three stochastic components in the model (movement, escalating, and confronts), the simulation results have a small standard deviation. Also because of randomness, the simulations have different number of turns before they end.

Figure 4.1 shows the results of one realisation, and Figure 4.2 shows the spatial distribution of the owners along a simulation. Clearly, the number of agents and the total amount of money of each group decrease along the simulation, and the more ambitious a strategy is, the sharper is the fall of the money and the number of agents following that strategy. Figure 4.1(c) also shows the mean value and the standard deviation of the experiments, plotted as arrows on the right side. There are some empty cells at the end of the simulation, which happens because, when there are only two agents in a cell, both might escalate, lose $10, reach the threshold, and decide to leave that cell.

The results show that the group following equilibrium has achieved most of the cells, despite the early misfortune shown in Figure 4.1(d). There are, nevertheless, some agents of other strategies at the end of the simulation. Equilibrium agents got the best results at the end, but they did not reached the majority by their own victories, it was indeed because the other strategies have lost their money faster. It is possible to see clearly that more aggressive agents destroy themselves rapidly and, therefore, following equilibrium yields a better chance of surviving. But, when there are a few aggressive agents in the simulation, they can avoid themselves and conquer some cells, justifying the growing number of cells conquered by $s_{0.5}$ at the end of the simulation. As the non-equilibrium strategies lose money faster than equilibrium ones, the initial money has a clear impact on the simulation, and agents following the equilibrium get more advantage of its increase. Simulations with higher values of initial money have shown that the difference between the number of cells of $s_{0.1}$ and $s_{0.5}$ becomes even larger.

Figure 4.3 shows the number of agents of each strategy that move during the first 150 turns of the simulation. More aggressive agents reach the threshold more frequently, until they start to leave the simulation. After the 30th turn, the number of movements decreases until the simulation stabilises.
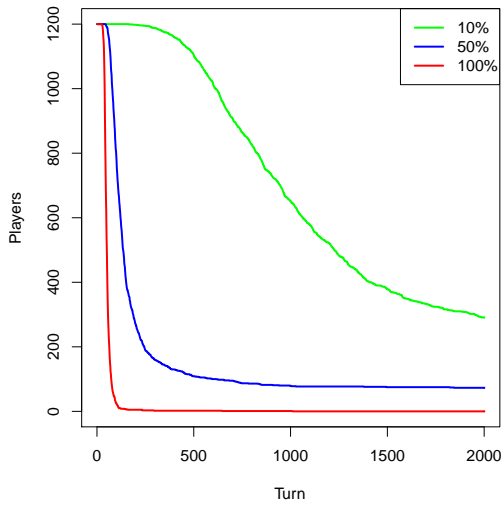
(a) Number of agents
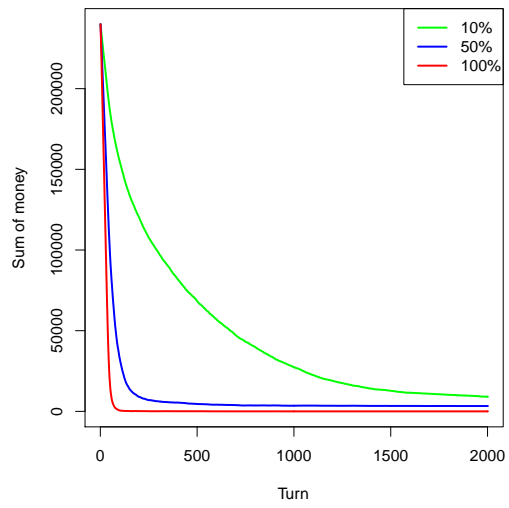
(b) Money by groups
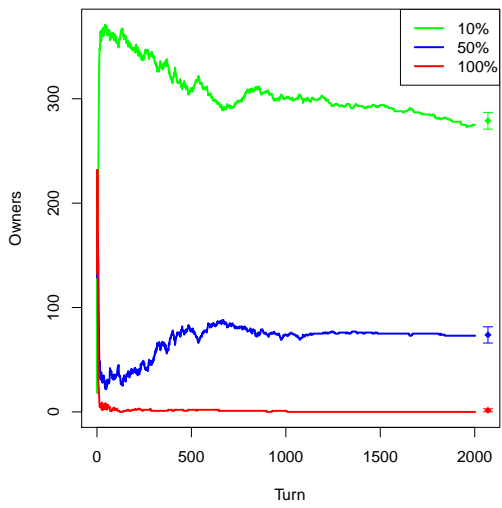
(c) Owners by groups
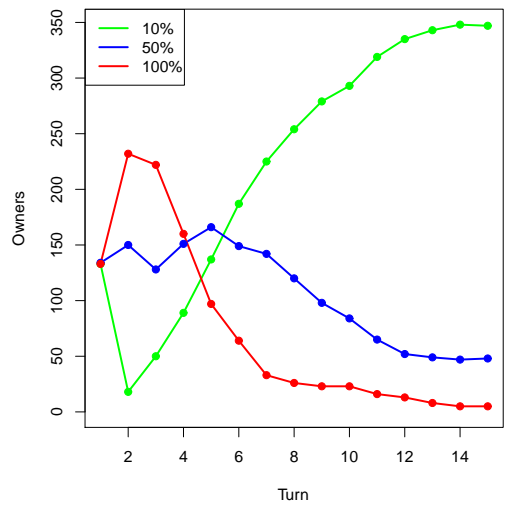
(d) Owners in the first 15 turns

FIGURE 4.1 - Results of the first experiment.

(a) Beginning of the simulation

(b) End of 1$^{\text{st}}$ turn

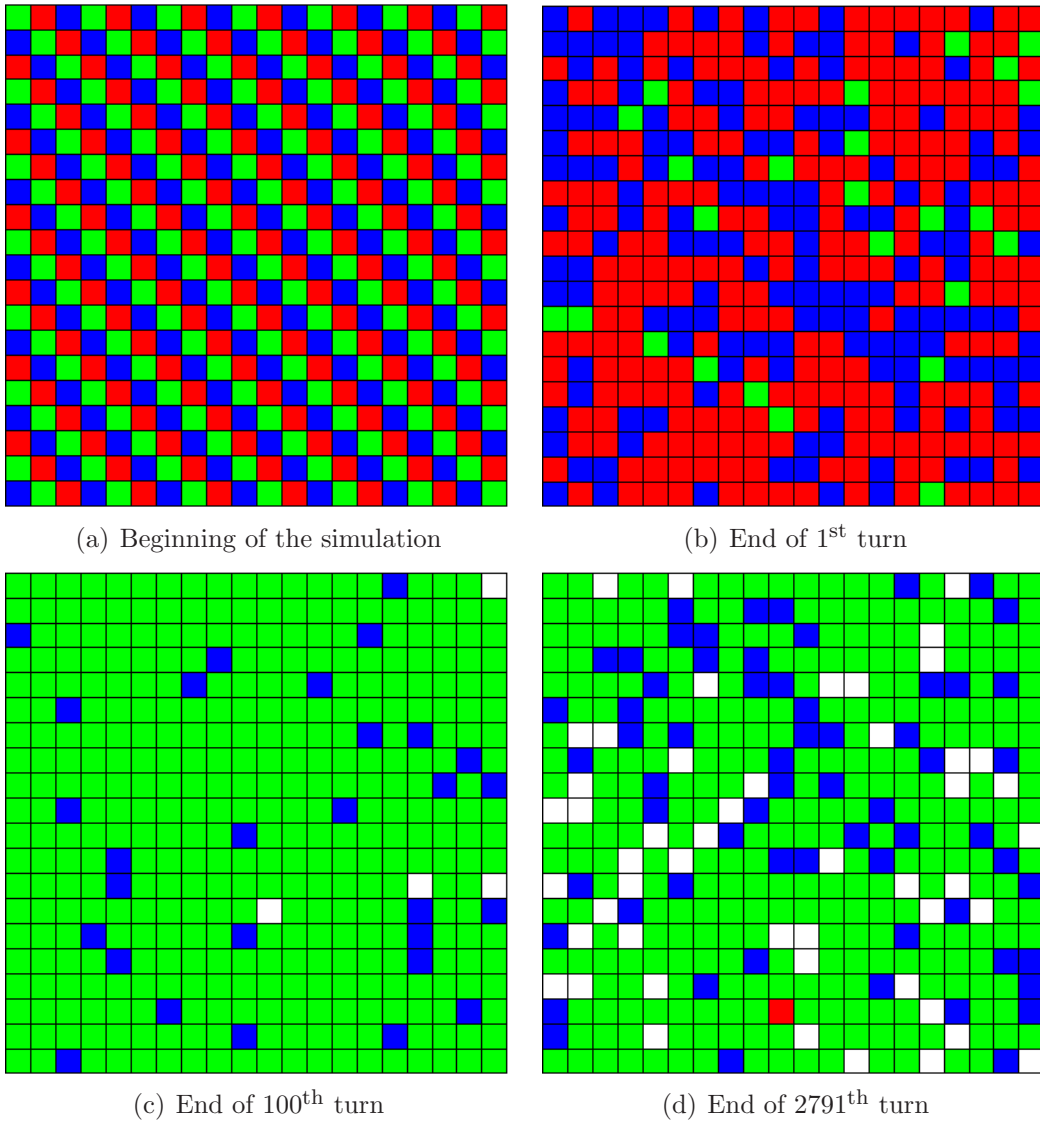(c) End of 100$^{\text{th}}$ turn

(d) End of 2791$^{\text{th}}$ turn

FIGURE 4.2 - Example of a run of the first experiment. Green cells are controlled by $s_{0.1}$, blue by $s_{0.5}$, red by $s_{1.0}$, and white cells are empty.
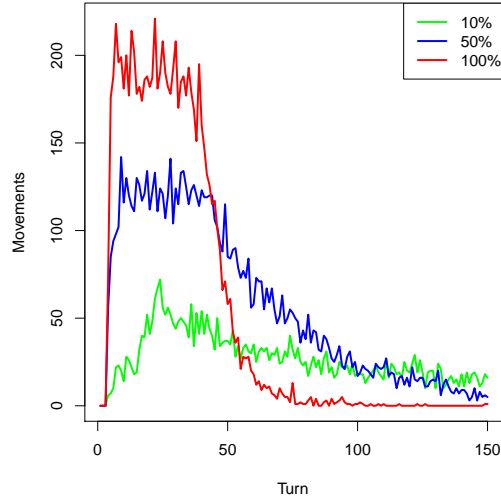
FIGURE 4.3 - Movements of each group in the first 150 turns.

4.6   Model variations

In M_0, Nash equilibrium is the best strategy for the competition against the other two chosen strategies. This section describes three other experiments, in order to verify whether the equilibrium strategy fares better also in other arrangements. In the first variation, we assign an infinite amount of money to each agent. In the second one, an extra amount of money is assigned to both agents after they play a game. We analyse how the simulations behave when money is less constrained in both experiments. In the last experiment, we use eleven strategies instead of only three.

4.6.1   Infinite amount of money

The initial amount of money can be large enough to keep all the agents alive until the end of the simulation. With it, the simulation will never converge to a stable state, because the agents will move indefinitely. We have the following model:

$$M_{inf} = (C_{20}, 2, \{E, \tilde{E}\}, u, \{s_{0.1}, s_{0.5}, s_{1.0}\}, 1200, \infty, 20).$$

As we can see in Figure 4.4(a), this simulation has a continuous repetition of the instability previously shown in the first 30 steps of Figure 4.3. This instability is favourable to $s_{0.1}$, which owns more than 85% of the cells, from the 30th turn until the end of the simulation, as we can see in Figure 4.4(b). The Figure also shows the

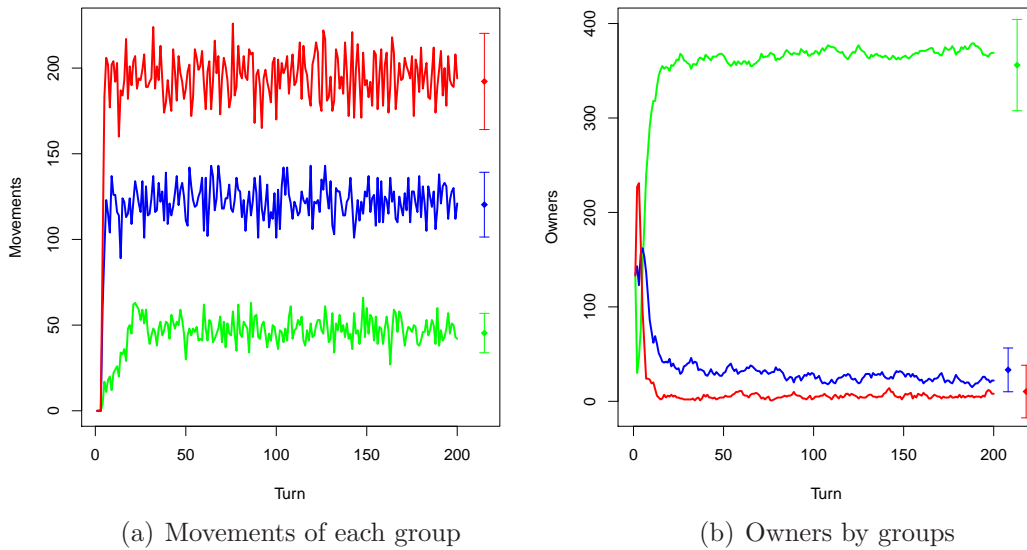(a) Movements of each group   (b) Owners by groups

FIGURE 4.4 - Model with infinite money.

average and standard deviation of each strategy.

Supposing a non-spatial environment, with agents meeting each other with equal probability, we can calculate the expected payoff of each strategy, deduce the number of movements, and compare it with the average number of movements of the simulation. The expected payoff of an agent is the average expected payoff against each group. The number of turns necessary for an agent of a given group to reach the threshold for moving is straight from this value, since the local satisfaction is $20. Then, as there are 1200 agents of each group, the average number of agents that would move each turn can be calculated. Table 4.3 shows these values, and in the lower part there is the difference between the expected movements in a non-spatial model and the average value of the movements in the experiments with infinite money. Clearly this value is less than in a non-spatial environment because each cell with an odd number of agents has one idle agent. Agents in space also reduce the expectations of a non-spatial environment because an agent that leaves a cell may find itself in a new one that happens to be more convenient. But this reduction is not equal for each strategy: the more an agent escalates, it can realise an unfavourable arrangement earlier, leaving the cell faster than the other strategies. It justifies why the decrease is proportional to the probability of escalating.

TABLE 4.3 - Impact of the escalating probability in the movement.

	$s_{0.1}$	$s_{0.5}$	$s_{1.0}$
Against $s_{0.1}$	-0.10	-0.10	-0.10
Against $s_{0.5}$	-0.90	-2.50	-4.50
Against $s_{1.0}$	-1.90	-5.50	-10.00
Mean	-0.97	-2.70	-4.87
Turns before an agent moves	20.61	7.40	4.10
Expected movements by turn	58.22	162.16	292.68
Movements with infinite money	47.25	123.20	196.13
Difference	10.97	38.96	96.55
Decrease (%)	18.84	24.02	32.98

4.6.2 Extra gain

The second variation still concerns the reduction of the number of agents along the simulation. But, instead of increasing the initial amount of money as in M_{inf}, we give an extra gain of capital to both agents at the end of each game, with the objective of keeping them alive. We describe a model with extra gain k as:

$$M_{g_k} = (C_{20}, 2, \{E, \tilde{E}\}, u + k, \{s_{0.1}, s_{0.5}, s_{1.0}\}, 1200, 200, 20).$$

As we increase all the payoffs by a constant value, the expected payoff of each agent also increases with this value, and thus the equilibrium point does not change (NASH, 1950). We explore six models with different values of extra gain to verify how it affects the results. They are: $M_{g+0.1}$, $M_{g+0.2}$, $M_{g+0.4}$, $M_{g+0.8}$, $M_{g+1.6}$, and $M_{g+3.2}$.

Figure 4.5 shows the number of agents in the six configurations at the end of 3000th turn. After that, the simulations have only some minor changes. The total number of agents that survive rises with the extra gain, but agents with a higher escalating probability have a lower tax of increase. Note that, contrary to the variation of infinite money, some agents are still being removed from the simulation because they reach the global threshold.

As the extra gain increases, agents that escalate less often stop reaching the threshold for moving and start to stand still, once they gain more money than lose. Figure 4.6 shows the owners in the six extra gains. The first strategy to lose mobility is $s_{0.1}$. From gain $M_{g+0.2}$ until $M_{g+0.8}$, it loses the majority to $s_{0.5}$. After gain $+0.4$, $s_{0.5}$ also starts to lose mobility, then cells, and finally in the simulation of $M_{g+1.6}$, it
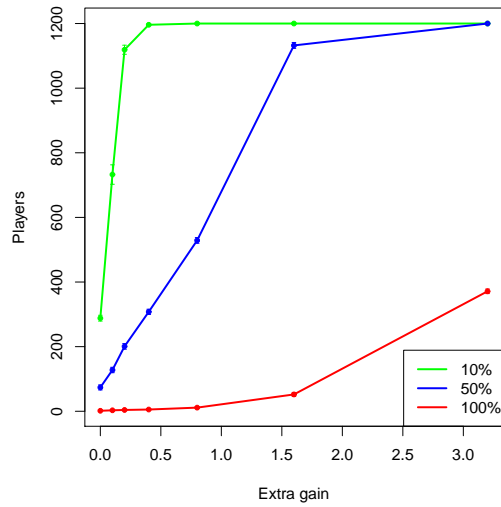
FIGURE 4.5 - Agents of each group in the simulations with extra gain after turn 3000.

already has lost most of the cells again to $s_{0.1}$. A bigger increase of extra gain does not lead $s_{1.0}$ to reach the majority because its agents have a strong disadvantage of self-destruction. With higher extra gain, the simulation becomes a set of local competitions without mobility, and therefore Nash equilibrium is the best strategy. Using the results we can infer that Nash equilibrium is the best strategy in the situations without mobility, but there is an interval of extra gain that can affect the agents's mobility, allowing other strategies to surpass Nash equilibrium. It happens because the Nash equilibrium of this game is not strict. If there is only an agent of $s_{0.5}$ or $s_{1.0}$ within a cell of only $s_{0.1}$ agents, then it can exploit the other agents, not by increasing its own payoff, but indeed by reducing the expected payoff of its opponents.

4.6.3 Eleven strategies

The third variation of the model explores a richer arrangement, trying to find out the best strategy for the first model. This model has agents following eleven distinct mixed strategies: $s_{0.0}$, $s_{0.1}$, ..., $s_{1.0}$. The other parameters are the same as in M_0. Therefore we have the following model:

$$M_{11} = (C_{20}, 2, \{E, \tilde{E}\}, u, \{s_{0.0}, s_{0.1}, \ldots, s_{1.0}\}, 1200, 200, 20).$$
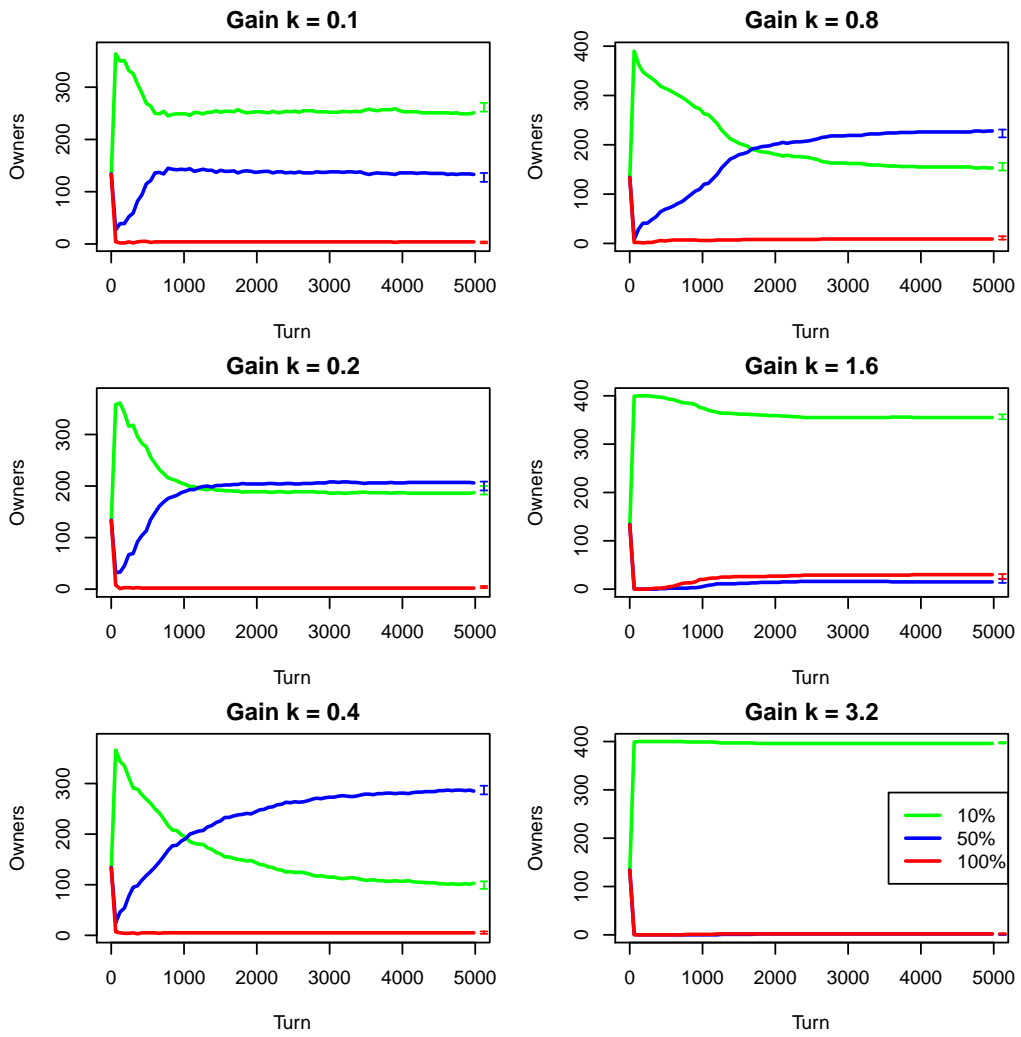
FIGURE 4.6 - Owners by group with six values of extra gain.

(a) Owners in the first 15 turns
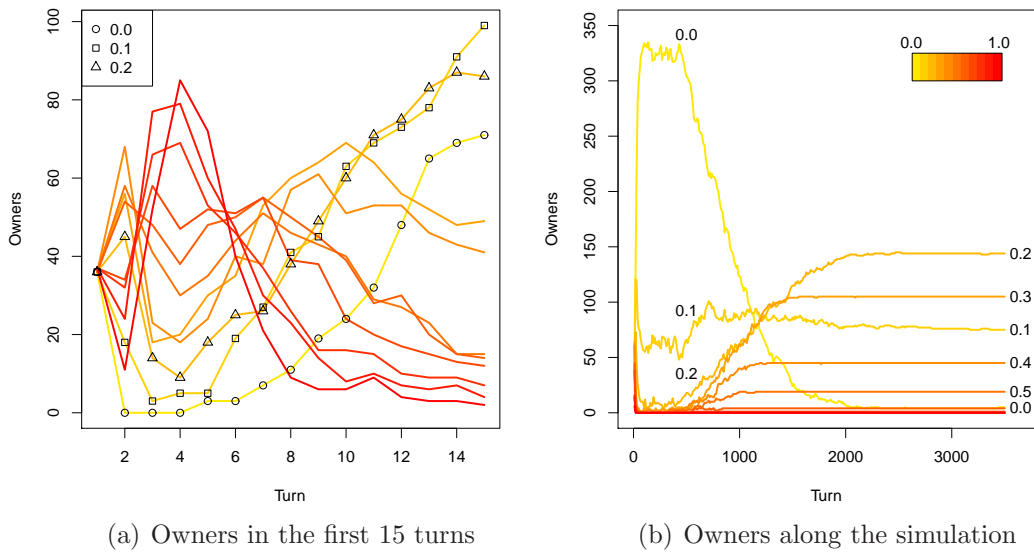
(b) Owners along the simulation

FIGURE 4.7 - Results of a single run with eleven strategies.

Figure 4.7 depicts the result of one experiment. In Figure 4.7(a), we can see that all strategies start with a similar number of cells, but quickly the ownership changes. Successful strategies in the first turns do not necessarily end the simulation with a high number of cells, as also shown in M_0. Some strategies are noteworthy. Agents that never escalate ($s_{0.0}$) quickly own almost all space, shown in the peak of Figure 4.7(b). However, as the simulation evolves, they lose most of them and, when the simulation finishes, they place at the 7th position. Equilibrium agents are quite successful, but they are in the third position at the end of the simulation. The second place belongs to $s_{0.3}$, and the strategy that achieves the best result is $s_{0.2}$.

Figure 4.8 summarises the cells each group owns. The red line points out the maximum number of cells each group has achieved among all turns of all simulations. The blue line shows the cells owned by each strategy at the end of the simulations, with mean and standard deviation. Strategy $s_{0.0}$ has reached a maximum of 366 cells, but this value was omitted from Figure 4.8 to give more emphasis to the other strategies. Note that the final ownership is similar to a gamma distribution. Also, there is no conflict in identifying the place of each group using the standard deviation as error, unless for groups from $s_{0.7}$ to $s_{1.0}$, which have achieved almost no cells.

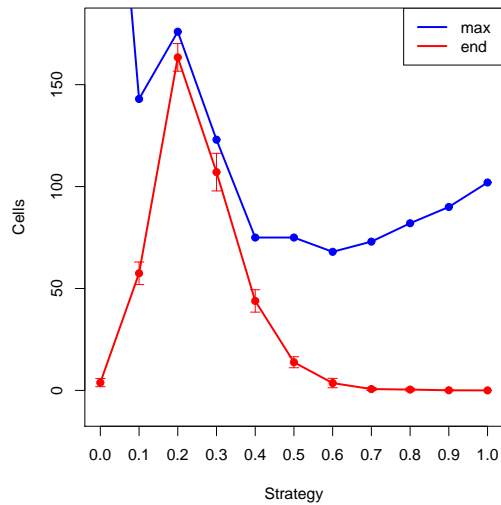We can roughly divide the simulation in three stages. In the first stage, from the 1st

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FIGURE 4.8 - Summary of the eleven strategies at the end of the simulations.

to the 5th turn, the aggressive strategies ($s_{0.5}$ to $s_{1.0}$) dominate the simulation and share the majority due to their initiative. The second stage emerges rapidly because they destroy themselves, making the purely cooperative strategy ($s_{0.0}$) arise until it reaches the majority around turn 20. Finally, in the third stage, when most of the aggressive agents have already left the simulation, the more successful strategies slowly but ceaselessly conquer the cells dominated by $s_{0.0}$ agents. But, in the end, some cooperative agents still remain. They can only survive due to the possibility of having empty cells, as described in the results of M_0. The behaviour of such an agent consists in waiting until the simulation ends or someone forces it to leave the cell. Although this strategy can be considered naive, the results show that this cooperative behaviour is nevertheless better than all the four more defective strategies ($s_{0.7}$ to $s_{1.0}$), and it almost draws with $s_{0.6}$.

The individual development of each strategy in all simulations is shown in Figure 4.9. The groups from $s_{0.8}$ to $s_{1.0}$ have a development similar to $s_{0.7}$. We can see that each strategy has similar results in all simulations. In the right part of each graphic, there is a vertical arrow showing the maximum and minimum number of agents of each strategy at the end of the simulations. Note that, in all strategies but $s_{0.0}$, the higher and lower points of the arrow match the maximum and minimum number of cells achieved by those strategies. Although there is an average of only 3 cells owned by $s_{0.0}$ at the end of the simulations, there is an average of 12 agents in
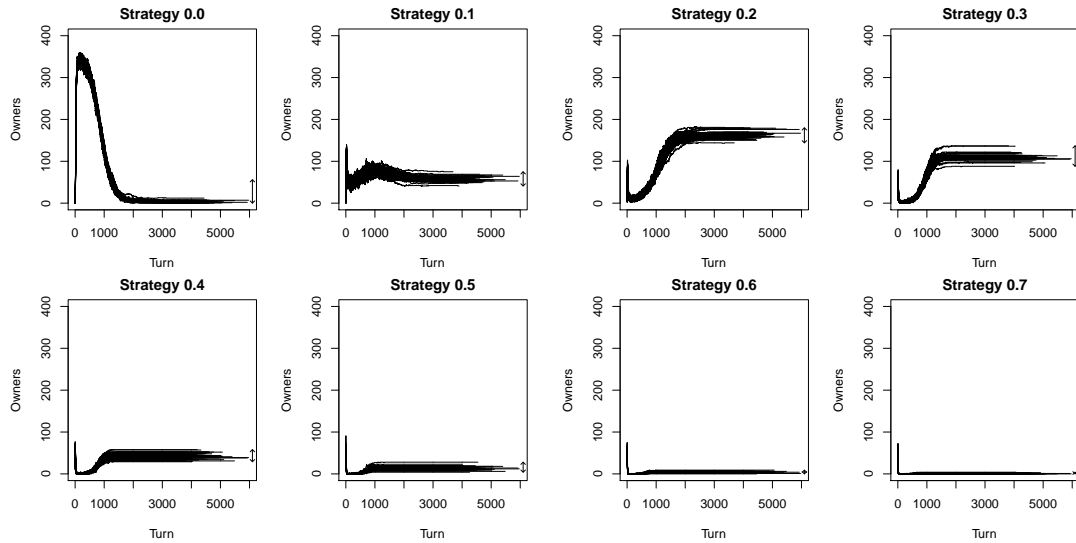
FIGURE 4.9 - Ownership of strategies along all simulations.

these cells, thus 4 times the number of cells owned by the group. Strategy $s_{0.0}$ is the only one that supports more agents than cells, something that may happen only within a purely cooperative behaviour, because agents of other strategies end up by escalating sometime as the simulation progresses. Therefore, when we talk about surviving instead of owning, this strategy gains one more position, surpassing $s_{0.6}$ and almost drawing with $s_{0.5}$. Thus, there is not much difference between never escalating and acting without any strategy ($s_{0.5}$), but it is better acting cooperatively than to adopt a tendency to escalate more often than not to escalate.

The majority achieved by $s_{0.2}$ and the second place achieved by $s_{0.3}$ can be explained by the result of M_{inf}, which states that, as an agent escalates more, it can realise a threatening arrangement earlier. The counterpart of escalating more is the higher destruction of agents within the same group. Nash equilibrium is the base for a stable relation, and it can be used as a starting point for the best strategy. In this model, the best strategy uses the equilibrium, but it adds some risk to get more mobility. The winning strategy mixes both characteristics; its agents are not exploited by threatening agents and can conquer cells from $s_{0.0}$ faster than equilibrium agents.

4.7   Conclusions

The results of the experiments show that changing parameters of the model that affect the mobility of the agents can lead to the success of strategies away from

Nash equilibrium. The results show that risky agents take more advantage of space, because they realise unfavourable arrangements earlier. But it may have a drawback of destructing players within the same strategy, which leads the strategy to be penalised as a whole. Therefore there exists a new equilibrium between these two factors. The evidence for this conjecture is shown in the results of M_{11}, presented in Section 4.6. In this model, there are eleven different strategies competing for space through a game with $s_{0.1}$ as equilibrium strategy, and the more successful strategy was $s_{0.2}$, winning each of the 50 runs. Although we do not expect the agents to be rational within this environment, Nash equilibrium is a good basis for the best strategy.

In all the models but the last one, there is not any grouping of agents at the end of the simulation because, if that occurs, the simulation has not ended yet. However, in the last variation of the initial model, it is possible to have more than one agent within each cell because we have one strategy that will never shoot ($s_{0.0}$). This leads to groups of agents following this purely cooperative strategy within the same cell, increasing the number of agents following this strategy at the end of the simulations.

Clearly, the models can be enhanced with further refinements and sensibility analysis. For example, all the models presented in this article have agents homogeneously distributed over the cells. In experiments not presented here, we studied models with the agents randomly distributed over the cells at the beginning of the simulation. The results are similar to the homogeneous case, with a straightforward increase on the standard deviation of the results, because there is a fourth stochastic ingredient, besides the escalating decision, the confronts, and the movement. We also tested the model using four neighbours by cell and also found similar results. In fact, these results have the same explanatory power as the models presented in this work.

We can cite some questions to be investigated in future works. An open issue is to explore evolutive models concerning population dynamics. New populations are generated from the fittest agents of the previous simulation, with some mutation in their characteristics. Will these populations competing for space evolve to Nash equilibrium? Perhaps, "we can only expect some sort of approximate equilibrium, since [. . .] the stability of the average frequencies will be imperfect" (NASH, 1950). Another question that can be investigated is the evolution of cooperation within the context of mobility. Clearly a pacific agent does not need to wait until losing $20 since it arrived at the cell to move to another cell. Therefore we would have

a more complex model, with the behaviour of agents differing not only by their mixed strategies, but also by their different ways to decide their actions based on the previous results. Within this environment, it is possible to investigate how the parameters of the model can affect how agents are grouped or repelled spatially, and the path taken by the different strategies.

4.8 Availability

The model presented in this work was implemented in the TerraME modelling framework (CARNEIRO, 2006). TerraME is a development environment for spatial dynamical modelling that links cell spaces to geospatial databases for data storage and retrieval. It uses the geoprocessing library TerraLib for working with geospatial data (CAMARA *et al.*, 2000), and Lua language to create models (IERUSALIMSCHY, 2003). Because of its simplicity, Lua has a large number of programmers in the game development community, an activity that has many requirements in common with social simulation. The source code as well as the programs necessary for running the simulations are available at `http://lucc.ess.inpe.br/doku.php?id=papers:mobility`.

5 POPULATION DYNAMICS, EVOLUTIONARY COMPUTATION, AND ADAPTABILITY IN GAMES ON CELLULAR SPACES[1]

Abstract

By using differential equations, evolutionary game theory shows that most of the games of competition for resources have equilibrium strategies named Evolutionary Stable. Although this approach can deduce these points, it is not possible to say how or whether a population may reach such equilibrium. In this work, we present an evolutionary agent-based model where individuals compete for space using mixed strategies. The objective is to investigate the evolution of populations competing for space through evolutionary rules. We study whether a population evolves and adapts itself to a stable state, the effects of each parameter in the development of the model, and how the attained equilibrium is related to the theoretical equilibrium. The results of the simulations show that populations are capable to converge to stable states, and can adapt themselves to new games along the simulation, corroborating the hypothesis that populations are capable to adapt themselves even if we relax the assumptions of analytical models.

5.1 Introduction

Much of the studies dealing with population dynamics in the last decades consisted in describing the reality through a set of mathematical equations and then deduce equilibrium points, also called stable states of the system. Due to its expressivity and elegance, this kind of modelling has achieved very useful results in different areas of science. Lotka-Volterra (LOTKA, 1925; VOLTERRA, 1926) as well as SIR

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[1]Part of this chapter is also published as two papers co-authored with A. M. V. Monteiro and G. Camara:

- *"Games on Cellular Spaces: An Evolutionary Approach."* Progress in Artificial Intelligence, 14th Portuguese Conference on Artificial Intelligence, EPIA 2009, Aveiro, Portugal. **Lecture Notes in Artificial Intelligence (LNAI)** 5816, pp. 535-546, 2009.

- *"Games on Cellular Spaces,"* **Second Brazilian Workshop of the Game Theory Society (BWGT): Accepted Papers**, 2010.

(KERMACK; MCKENDRICK, 1927) equations are examples of successful approaches for dealing with complex phenomena of predator-prey dynamics and disease propagation, respectively.

In fact, this way of modelling rely on some unreal assumptions which are too complicated to be relaxed, such as the absence of spatial structures and homogeneity. In the same way much of science that worked with system dynamics was based on mathematical models of differential equations in the past, today it has been turning into computation based models, which allow relaxing the *hard* assumptions of mathematical models. By explicitly simulating the individuals involved in the system through agent-based modelling, we can find out useful, flexible, and probably generic models, whose original equilibrium deduced with equations may turn into a special case of the model, enhancing our understanding of complex systems.

A well-known work that uses mathematical models is the one from Maynard Smith, which investigated the relations between evolution and the theory of games (MAYNARD SMITH, 1982). He showed that Game Theory can be used to study species competing for resources. His evolutionary theory demonstrates that there exist equilibrium points named Evolutionary Stable Strategies (ESS), which means that, if all members of a population adopt the same ESS then no mutant strategy could spread under the influence of natural selection.

However, this model has some artificial assumptions, such as the fact that the population is infinite, and that every player of the model competes with each other. Moreover, although this approach can deduce equilibrium points, it does not say how or whether the population could attain such equilibrium. Even Maynard Smith has admitted that "[a]n obvious weakness of the game-theoretic approach to evolution is that it places great emphasis on equilibrium states, whereas evolution is a process of continuous, or at least periodic, change" (MAYNARD SMITH, 1982). Moreover, it has been shown that some equilibrium points, although stable, may not be attainable (EPSTEIN; HAMMOND, 2002). Therefore, we need to emerge these patterns in a generative way, which can not be done by using replicator equations.

Within this context, this work studies two questions: *What would happen with the evolution of a population if we relax the assumptions stated by the model of Maynard Smith? How can a population reach such equilibrium?* We investigate both questions by working with a model that is based on two main features: *evolutionary compu-*

tation and *space*. Evolutionary computation emerges as a natural way of building models where there are changes in the composition of populations, mainly by sending somewhat codified information from generation to generation. In the other side, the role of spatial self-structuring in the study of games has drawn a lot of attention in the last years. Particularly, games with spatially explicit factors have proven to be useful for modelling biological and economic environments (NOWAK; SIGMUND, 2000). This approach has two main advantages. Firstly, it relaxes the assumption that every agent has to compete with each other, because the locations can determine with whom each agent will interact. Secondly, putting agents into spatial partitions allows us to represent the limited capacity of the environment, forcing the population to have a maximum size.

There are some empirical evidences in the literature showing that populations compete for space because their members need to guarantee food, which can decide their fate. An experiment showed that owning space may lead to survivor is presented by (RIECHERT, 1981). The author describes the percentage of diary corporal weight change of spiders. There are two groups of spiders: one that owns some territory and another that does not. Most of spiders that do not own a territory lose weight, while almost all owners increase their weight, which leads to a greater chance to survive. There are also examples of species belonging to other animal classes, such as turtles (STICKEL, 1950) and birds (LACK, 1966).

In order to deal with these relaxed assumptions, we present an agent-based model to investigate the stability and adaptability of populations competing for space through evolutionary rules. We study whether a population evolves and adapts itself to a stable state, the effects of each parameter in the development of the model, and how the attained equilibrium is related to the theoretical equilibrium.

This paper is written as follows. In Section 5.2, we present some game theory concepts and related works. The evolutive model is described in Section 5.3, and the results of the simulations are presented and analysed in Section 5.4. Finally, the conclusions and future works follow in Section 5.5.

5.2 Games and evolution

In a n-players game we have, for each player:

a) a finite set of pure strategies (actions);

b) a payoff function, mapping all n-tuples of pure strategies to real numbers.

One mixed strategy is a collection of non-negative numbers adding up to 1, corresponding to probabilities of using each of the pure strategies. The mixed strategy defines the tendencies of a player. Each time it plays, it will choose randomly one of its pure strategies, based on the probabilities defined by the mixed strategy.

For example, let us take the chicken game. Two players have the choice to escalate or not to escalate a brawl. Each one has no access to the decision of the other before taking its decision. If none of them escalates, nothing happens. If only one escalates, the other player runs away, and the winner receives 1 from the other. But, if both decide to escalate, each player pays 10 due to medical care. This game is said to be symmetric, because both players employ the same pure strategies and payoffs, as we can see in Table 5.1. Given that this game has only two pure strategies, we say that s_x, $0 \leq x \leq 1$, is the mixed strategy of escalating with probability x.

TABLE 5.1 - Chicken game payoffs, in pairs (A, B).

	B escalates	B does not escalate
A escalates	$(-10, -10)$	$(+1, -1)$
A does not escalate	$(-1, +1)$	$(0, 0)$

From this description for a non-cooperative game, Maynard Smith presents his evolutionary model, based on the following assumptions:

a) infinite population;

b) asexual reproduction;

c) pairwise contests taking place between two opponents;

d) symmetric contests, that is, opponents do not differ in any way discernible to themselves before contest starts;

e) finite set of alternative strategies.

The last assumption fits in with the description of a game presented previously, assumptions (c) and (d) are restrictions over the description above, while the two first assumptions are required to make the model mathematically tractable.

Maynard Smith's model is based on replicator equations, one for each strategy, describing what will happen with the population given the proportions of each strategy in the previous time step. With it, individuals play against the average strategy of the population. They reproduce by passing their strategies to a new generation, according to their biological fitness, which is represented by the payoffs. Although not explicitly described on the model, it is imagined that alternative strategies of the game may occur through a process like mutation.

The stable state of the replicator equations is the equilibrium point of the game, which is called ESS. If the whole population adopts this strategy, then there is no better strategy for a single member than also adopting it. Consequently, an ESS is considered resistant against any mutant strategy that may occur in the population.

An ESS is a refinement of a Nash equilibrium. Let a two player game, with $p(a, b)$ representing the payoff for playing strategy a against strategy b. A given strategy a is a Nash equilibrium if and only if:

$$p(a, a) \geq p(b, a), \forall b \neq a.$$

The main critics related to this definition of equilibrium is that there may exist a neutral strategy b that scores equally well if compared to a, which could make the population deviating from the equilibrium point. Because of that, Maynard Smith defines more restrictive conditions that would be required for a given strategy a to be an ESS. They are:

a) $p(a, a) > p(b, a)$, or

b) $p(a, a) = p(b, a)$ and $p(a, b) > p(b, b), \forall b \neq a$.

Maynard Smith divides Nash equilibrium in two parts, related to the possibility of a neutral strategy to emerge in the population. In the case (a), there is no possibility for such strategy because the equilibrium point is said to be *strict*. In the case where this strategy may emerge (b), the members who continue to play the equilibrium strategy have a winning advantage when playing against the neutral strategy.

Two possible interpretations for the ESS are pointed out by Maynard Smith. The first one refers to mixed strategies, meaning that each member of the population follows the same equilibrium mixed strategy. In the case of the chicken game, the whole population would follow $s_{0.1}$. The second solution is related to pure strategies. We have players following different pure strategies in such a way that the average strategy of the overall population is at equilibrium. Again with the chicken, a stable population has 90% of its members following $s_{0.0}$ and 10% following $s_{1.0}$.

A couple of works in the literature propose simulation models to relax some assumptions of the mathematical model. Orzack and Hines show that the probability of a population evolving to an ESS is proportional to its size, needing to have large populations to get closer to the theoretical equilibrium (ORZACK; HINES, 2005). Fogel and others (FOGEL *et al.*, 1997; FOGEL *et al.*, 1998) describe models with populations starting at ESS but, instead of staying stable, the populations evolve to other strategies and never stabilise, corroborating to the hypothesis that a population is capable to evolve to ESS only if it is infinite. Figure 5.1 shows an example of their results.

However, in both cases, there is no representation of space within the model, and therefore each agent still competes with each other. Moreover, there is not even a genetic dimension. It means that, if the population starts without any member following a given strategy, this strategy will never emerge in the model. Because of that, these models are highly dependant on the initial population.
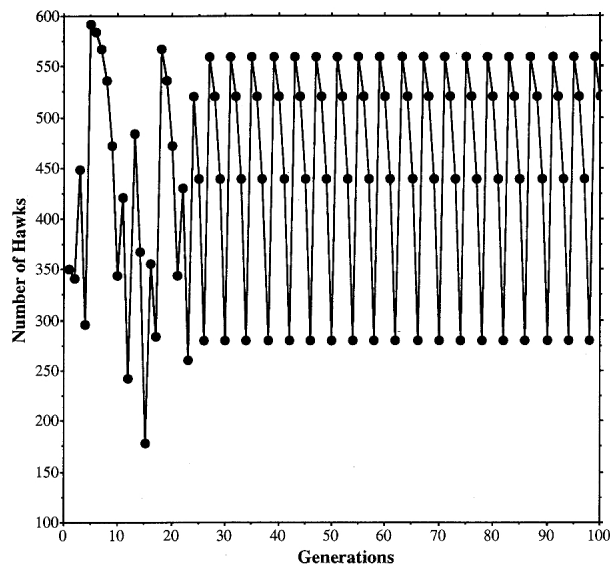

FIGURE 5.1 - The instability of ESS in small populations with no representation of space. Source: (FOGEL *et al.*, 1997).

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By adding space as a new dimension for studying these competitions, we can directly relax two assumptions: (i) agents having to compete with each other and (ii) working with infinite populations. However, much of the simulation studies that add space as a fundamental component of their models are interested in the evolution of cooperation using only pure strategies or meta-strategies that use the last results to choose the next action (NOWAK; SIGMUND, 2000). Only the strategy spreads over the space because the agents may change their behaviours but not their spatial locations. In some cases, the agents have little mobility, mainly by waiting for an empty cell to move (EPSTEIN, 1997). In this work, we study the evolution of a population that compete for space using mixed strategies, which makes more feasible to apply concepts of evolutionary computation, because in this context we have a space of strategies to be searched, instead of only a small number of pure strategies. The model is presented as follows.

5.3 The model

The approach of this work is based on the non-evolutive model presented on Chapter 4. As we are working with population dynamics, we need to transpose some assumptions of the original model to this new environment. The basic idea is that the *fitness* of an agent is materialised as the metric for its satisfaction. Local satisfaction represents the maximum effort one can dispend when competing for a cell and global satisfaction denotes the initial fitness. The model starts with each agent having the same fitness and the same mixed strategy. The game represents the way by which agents solve the conflict for space partitions, and the results affect directly their fitnesses. Mixed strategies indicate the level of aggressivity one agent has. This property is passed from generation to generation, but one agent can never change its own strategy.

The competition executes in the same way of the basic model, until it reaches a stable state, when there is at most one agent in each cell. After that, a reproductive part of the model takes place. The agents that own cells at the end of a competition deserve to pass their strategies to the next generation, independently of their final fitnesses. Therefore, the model considers a competition as one evolutive step, a *survival of the fittest*.

Once a competition finishes, each agent that has conquered a cell produces a couple of descendants within that cell. The agent tries to transfer its own strategy to the

offsprings, but their learning ability is limited. Because of that, they may have a strategy slightly different from the one of the father, according to a fixed mutation probability. Whenever this mutation is activated, the offspring inherits the strategy of its father with some random change; otherwise it takes exactly the same strategy. Whether the mutation will take place and the change in the father's strategy are both calculated independently for each offspring.

Each father leaves the model right after it generates its descendants. This leads to non-overlapping generations, which is a common approach in the literature (FOGEL *et al.*, 1998; MAYNARD SMITH, 1982; ORZACK; HINES, 2005). Finally, the evolutive process can be repeated indefinitely, depending only on the initial population of the first generation.

The model is relatively complex and has five stochastic ingredients, three of them coming from the original model, and the other two added by the evolutive part. They are:

a) select pairs of agents within the same cell to play,

b) choose a pure strategy based on a mixed strategy,

c) relocate, once the agent is unsatisfied,

d) mutate or not a given strategy to a descendant, and

e) choose the mutation change, once it is activated.

The three first random elements come from the original model, and the other two are added by the reproductive part of the model. Random numbers with probabilities associated to each possibility are used to generate values in (b) and (d), the other three are simply samples from finite sets.

The simulated model takes place in a cellular space represented by a grid, with possible movements of an individual being almost to four neighbours (up, down, left, and right). Cells on the edges have only three alternatives for movement, and cells on the corners have only two. In the simulations, we chose the parameter values shown in Table 5.2. These values do not change unless specified in the tests. The initial population of the model is composed by three agents following $s_{1.0}$ in each

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of the 400 cells. Note only that, with a mutation change of ± 0.1, we allow eleven possible strategies to arise: $s_{0.0}, s_{0.1}, ..., s_{1.0}$.

TABLE 5.2 - Parameter values of the evolutive model.

	Parameter	Value
Non-evolutive part	Cellular space size	20×20
	Game	Table 5.1
	Initial fitness (global satisfaction)	200
	Local satisfaction	20
Evolutive part	Mutation probability	1%
	Initial strategy	$s_{1.0}$
	Mutation change	± 0.1
	Number of descendants	3
	Generations	300

5.4 Simulations

This model was implemented in TerraME[2], a software environment for spatially explicit dynamical modelling (CARNEIRO, 2006). It supports different modelling paradigms, with agent-based modelling being one of them.

Although the model has a couple of random ingredients, repeating a simulation with the same parameters have produced similar developments. Because of that, the results presented in this section are only single runs of the model.

5.4.1 Convergence

The initial test for studying this model tries to answer the following question: *Is it capable to converge to a stable state?* We simulate the model with all parameter values as previously described. Figure 5.2 shows the number of cells owned by each of the eleven strategies at the end of each generation, with Figure 5.2(a) zooming the first 60 generations.

The model starts with $s_{1.0}$ filling the *whole* cellular space, but it cannot maintain this situation, falling to around 75% of the cellular space (approximately 300 cells). This happens due to two facts. First, two agents within a cell may escalate, reach the threshold, and relocate, leaving the cell empty. Second, as the population is highly

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[2]http://www.terrame.org
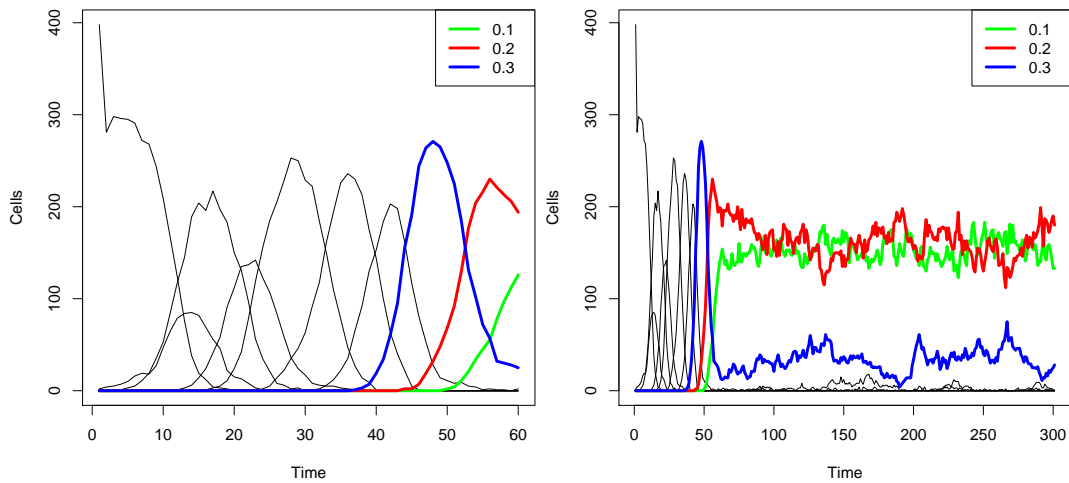
(a) First 60 generations.   (b) All the simulation.

FIGURE 5.2 - Development of strategies in a simulation with population of $s_{1.0}$.

homogeneous, the fitness of every agent is reduced in the same way, enhancing the chance that two agents kill themselves while competing for a cell.

When the first mutation to $s_{0.9}$ arises in the population, it spreads itself over the cellular space. This strategy can then produce agents following $s_{0.8}$, then $s_{0.7}$, and so on, some of them reaching the majority of cells and some not, until $s_{0.1}$ appears, as shown in Figure 5.2(a). But $s_{0.1}$ cannot get rid of $s_{0.2}$ and both compete indefinitely. Moreover, $s_{0.3}$ also conquers a significative part of the cellular space, as we can see in Figure 5.2(b).

The results of the simulations agree with the ones of Chapter 4, that state that as more frequently an agent escalates, greater is its ability to realise cells with a higher number of agents that also escalate frequently because it will reach its threshold for moving faster than the ones that escalate less. Moreover, realising threatening environments earlier is important because the agents have limited fitness and need to stay alive. This leads to an increase in the surviving chance of sub-optimal strategies such as $s_{0.2}$ and $s_{0.3}$. Therefore, although there is a single theoretical equilibrium point, there is no single best strategy in this competition for space.

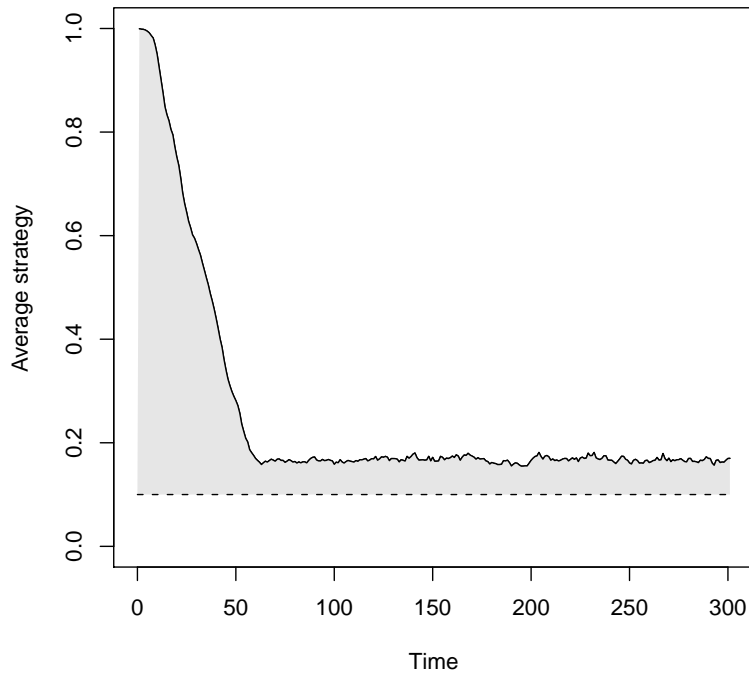Although neither strategy can surpass all other strategies and dominate the overall

FIGURE 5.3 - Convergence of the average strategy with population of $s_{1.0}$.

population, an evolutive process indeed occurs, differently to what occurred in the models of the literature that relax some assumptions stated by Maynard Smith's model but do not have a spatial representation. If we calculate the average strategy of the population by summing up the mixed strategies of every owner at the end of a generation and divide by the total number of agents that own a cell, we can verify that the population indeed converges to a stable state. In Figure 5.3, we have the average strategy of the population, with a dashed line showing the theoretical equilibrium point[3] ($s_{0.1}$). The average strategy converges to a stable state of around $s_{0.165}$ before the 70th generation and stays relatively close and above the theoretical equilibrium point for more than 200 generations, until the simulation ends. Moreover, the convergence occurs without any exogenous effect forcing the population to move towards the equilibrium point.

The result of this model is a mixing of the two definitions of equilibrium described by Maynard Smith. In the simulation results, we do not have the overall population following the same equilibrium strategy, nor agents following only pure strategies, but what really happens is a combination of mixed strategies whose average is a

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[3]In all figures of this chapter, dashed lines represent the theoretical equilibrium point.

stable equilibrium point. In fact, in both equilibrium interpretations described by Maynard Smith, the average strategy is equal to the theoretical equilibrium point.

In the next subsections we execute an exhaustive test on the parameters of the model, in order to verify its robustness and understand why the average population had presented that difference to the theoretical equilibrium point. We use a *ceteris paribus* methodology, fixing all but one parameter in different groups of simulations. There are two reasons for choosing this strategy. First, it helps us to identify how parameters can affect the evolution and stability of the model. Second, as we have a set of reference values of parameters from the analytical model (which are basically zero or infinite), allowing us to analyse how the model behaves when the parameters evolve towards these analytical values.

5.4.2   Initial population

The first results cannot enlighten whether the evolutive process always happens because it uses only a single initial population. To test the evolutionary capability of the model, we need to examine different initial populations. We test four different arrangements with the whole population following the same strategy s_k, where $k \in \{0.0, 0.1, 0.5, 1.0\}$. That is, the initial population of each model is composed by a single strategy, which are: never escalate, equilibrium, random, and always escalate.

Figure 5.4 contains the number of agents following each strategy at the end of each generation, with the three most successful strategies ($s_{0.1}$, $s_{0.2}$, and $s_{0.3}$) as coloured lines. These strategies have similar development in the four cases, with $s_{0.1}$ and $s_{0.2}$ competing indefinitely for the first place, but none of them surpassing the other two and dominating the whole population, similarly to the previous results. Figure 5.5 shows the average strategy of the overall population in each simulation. The four simulations converge to the same stable state, in spite of the initial arrangement of the models. The only difference between the models is the number of generations necessary to converge to the stable state, which is proportional to the distance of the initial strategy to the equilibrium state.

As we have tested the model with the extreme initial populations, $s_{0.0}$ and $s_{1.0}$, and the model can generate other strategies through mutation, we can infer that the population is capable to converge to a stable state with whatever initial population, even a heterogeneous one. Moreover, although there are five random ingredients,
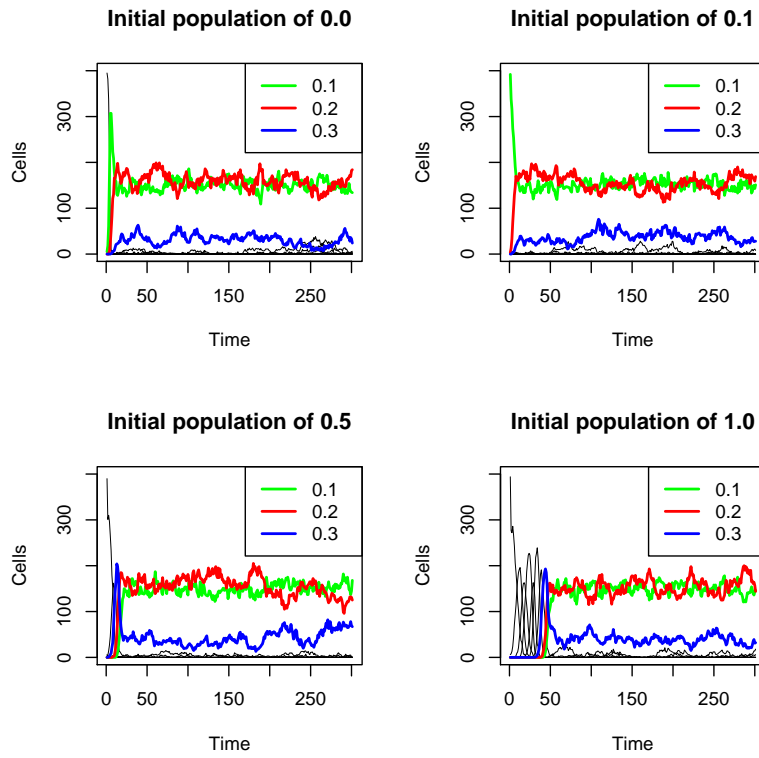
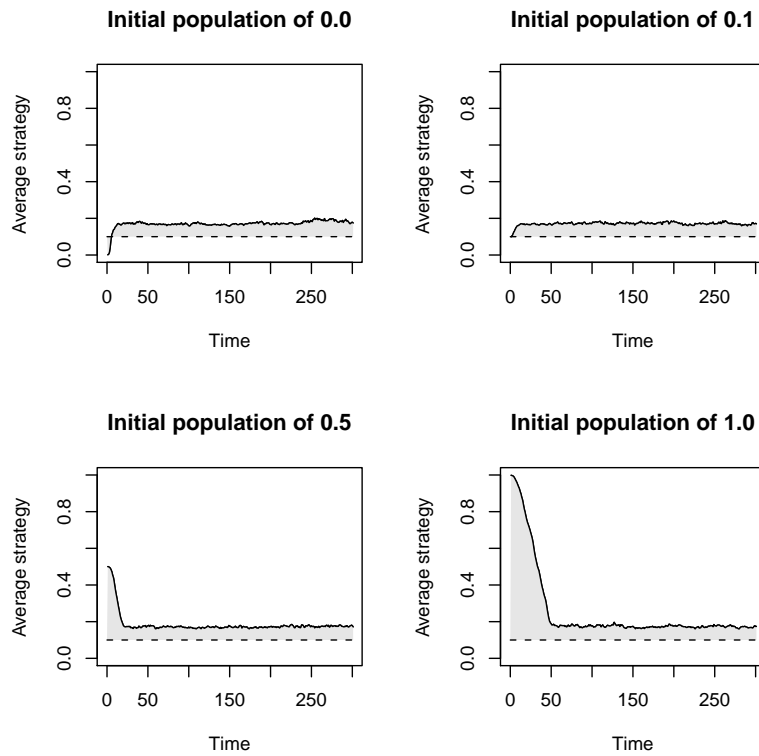FIGURE 5.4 - Evolution of strategies using different initial populations.

FIGURE 5.5 - Average strategy using different initial populations.

their effects do not make the model chaotic. Therefore, the model presented here is more robust than the others in the literature because it does not depend on the initial population.

5.4.3 Mutation probability

As presented in the last results, the model may require more or less time to converge, depending on how far the initial strategy is from the equilibrium state. Another parameter that affects the convergence time is clearly the mutation rate. This subsection analyses the effects of this parameter. Four simulations were carried out, each one with a single mutation rate, varying from 0.1% to 50%. It is not interesting to have the mutation rate greater than 50% because, in this case, most of the descendants would follow a strategy different from their predecessors.

It is intuitive and the results show that increasing mutation leads to a faster convergence to a stable state, as we can see in Figures 5.6 and 5.7. The former shows the development of the individual strategies along the simulation and the latter shows the average strategy. With mutation of 0.1%, the simulation takes more than 150 generations to converge, while with 50% it converges in less than 25 generations.

However, mutation probability induces other impacts beyond convergence time. First, the distance between the average strategy and the theoretical equilibrium is proportional to the mutation rate. It is possible to see it clearly in Figure 5.8, which summarises the average population after the 200th generation in eight different simulations. Note that the "mutation probability" axis is in logarithmic scale. Our conjecture is that, when the mutation probability tends to zero, the population also tends to the theoretical equilibrium. But note that the mutation can never be zero because we would have a model that is not capable to change its population and evolve. Therefore the mathematical concept of limit fits in with this situation.

Second, if we compare the models after they converge to a stable average strategy, each individual strategy reduces its oscillation as the mutation increases, stabilising in a well-defined small interval, with mutation equals to 50%. That happens mainly because, with higher mutation, if one strategy is close to a very successful strategy, it guarantees a minimum number of agents by inheritance. For example, with mutation 50%, $s_{0.3}$ inherits 25% of the descendants of $s_{0.2}$. The increase in the mutation rate makes possible the emergence of agents following almost each possible
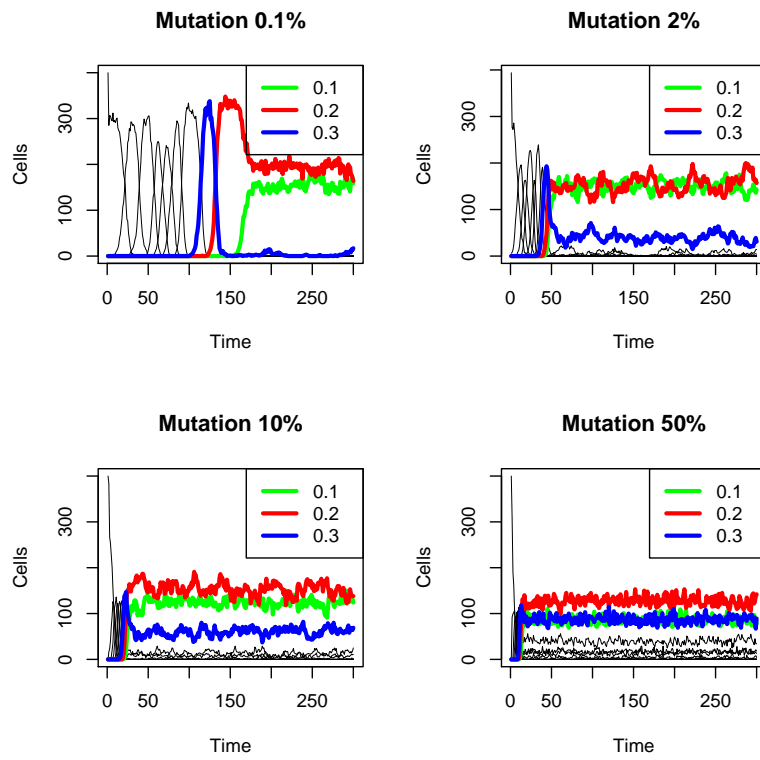
FIGURE 5.6 - Distribution of individual strategies with different mutation probabilities.
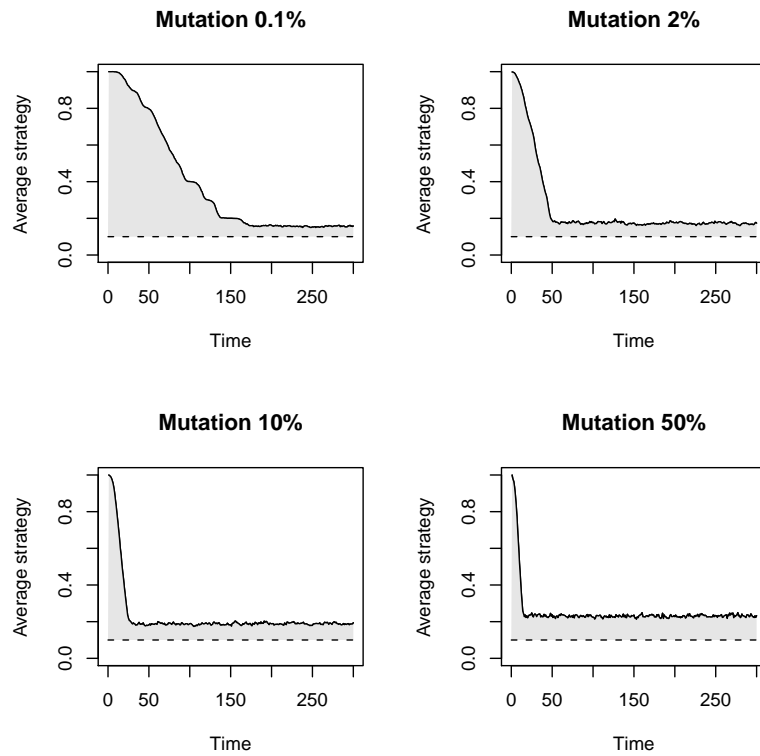

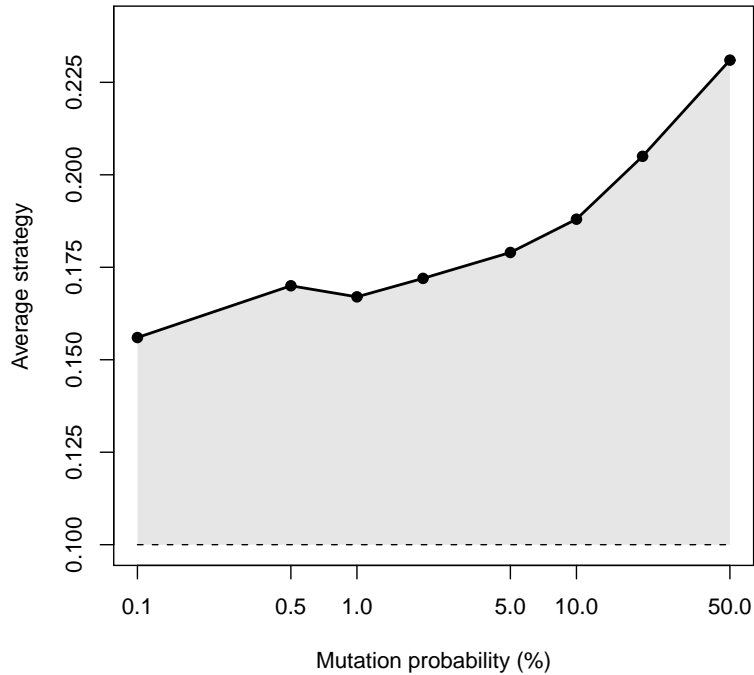FIGURE 5.7 - Average strategy with different mutation probabilities.

FIGURE 5.8 - Final average strategy of the population with different mutation probabilities.

strategy. Consequently, these few agents away from equilibrium increase the average strategy of the model significantly.

Third, the distribution of individual strategies makes the model converge towards a discretised gamma distribution as mutation chance increases. This result can be seen in Figure 5.9, where $s_{0.1}$ decreases and $s_{0.3}$ increases as mutation increases, until they almost draw when we have mutation of 50%. Note that, because of the higher mutation probability, the development of strategies away from the theoretical equilibrium makes the overall distribution smoother.

Finally, the mutation rate is proportional to the total number of agents that survive at the end of each generation. Figure 5.10 shows the number of agents that give rise to descendants along the four simulations. The four simulations have a minimum of around 75% of the cellular space in the early generations, just like the initial results in Figure 5.2. After that, the number of agents grows and stabilises. In each graphic, there is a dotted-and-dashed line pointing out 90% of occupation of the cells to facilitate a visual comparison. With mutation 0.1%, the population stabilises below this line, while with mutation 10% or more the average number
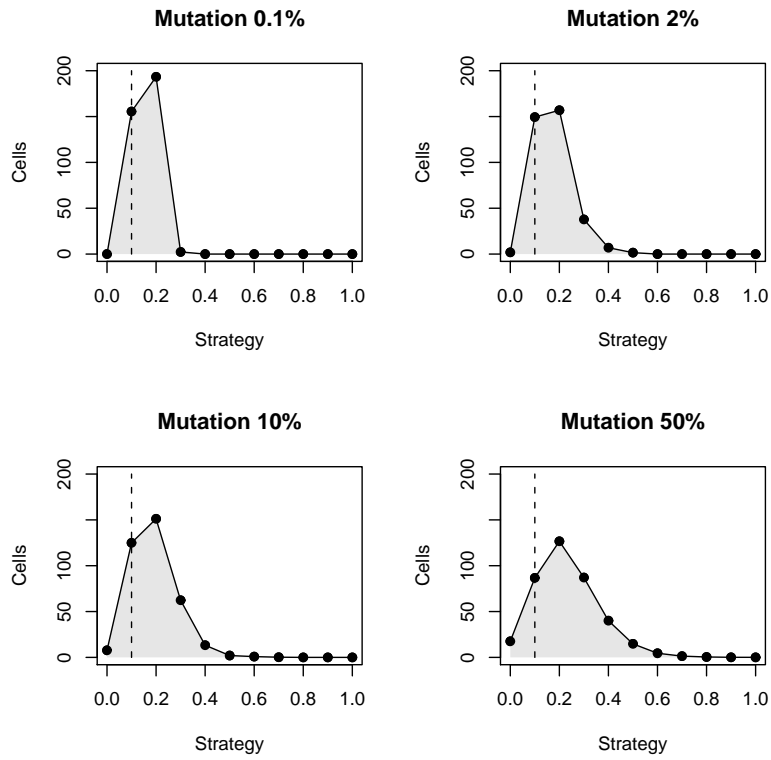
FIGURE 5.9 - Average number of agents of each strategy after convergence, with four mutation rates.
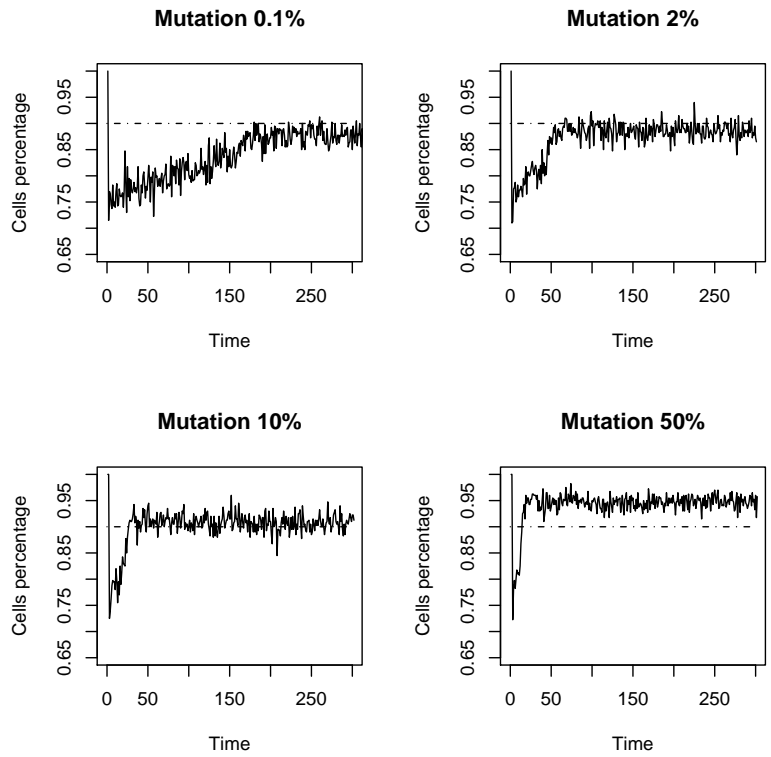

FIGURE 5.10 - Effects of mutation on the number of occupied cells at the end of each generation.

of cells surpasses the line. A greater diversity in the number of strategies allows a better occupation of the cellular space at the end of each generation. As there are more agents with different strategies with the increase in the mutation, the fitnesses are reduced more heterogeneously, which leads to a lower probability of two agents removing themselves from the model simultaneously.

5.4.4   Mutation change

The second parameter concerning mutation is the amount of change once it is activated. The value used in all previous simulations was ± 0.1. If we reduce the mutation change, we allow more and more strategies to emerge, instead of only eleven.

Figure 5.11 shows the average strategy of the population after the model stabilises in simulations with six different mutation changes, from ± 0.0005 to ± 0.1. Note that the "mutation change" axis is in logarithmic scale. With mutation change as lower as ± 0.0005, an agent that inherits a strategy with mutation will escalate once more (or less) in each 2000 turns on average if compared with another agent that inherits a strategy without mutation. The simulation of this case gave rise to the average strategy $s_{0.135}$, meaning that each generation would long approximately 1000 turns. Therefore, the agent would escalate once more (or less) in each two generations, making the competition between these two agents almost a random drift. However, as the model evolves and new mutations arise, it forces the average downwards.
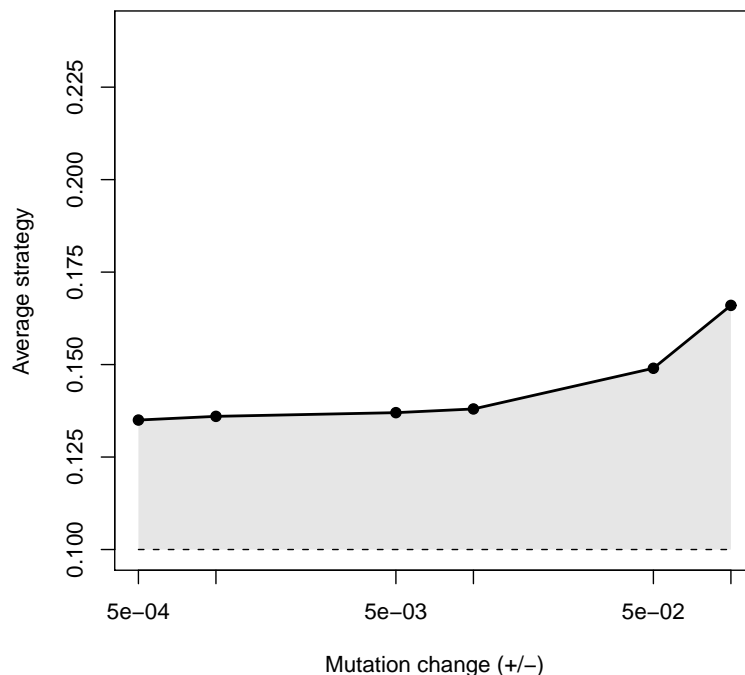

FIGURE 5.11 - Final average strategy of the population with different mutation changes.

We can see that mutation change has the same effect of mutation probability: when it goes towards zero, it makes the model converging to a point closer to the theoretical equilibrium point. As in the mutation probability results, this value can never be zero, otherwise the model could not evolve.

5.4.5   Initial fitness

The initial fitness of every agent in all previous simulations was 200. As it represents the resources the agents have to compete, changing it affects directly the number of turns of each generation. In this subsection, we analyse its effects on the evolution of the population using twelve different values for initial fitness, ranging from 50 to 102400 (200×2^{-2} to 200×2^9).

Figure 5.12 shows the average strategy of the population after the model stabilises in the different simulations. Note that the "initial fitness" axis is in logarithmic scale. We can see that there is a maximum distance to the theoretical equilibrium point, when the initial fitness is 1600. However, this is point may be less or more, depending on the configuration of the parameters. Indeed, the result that worth to be noted is that there is a tendency towards $s_{0.1}$ as initial fitness grows towards infinite.
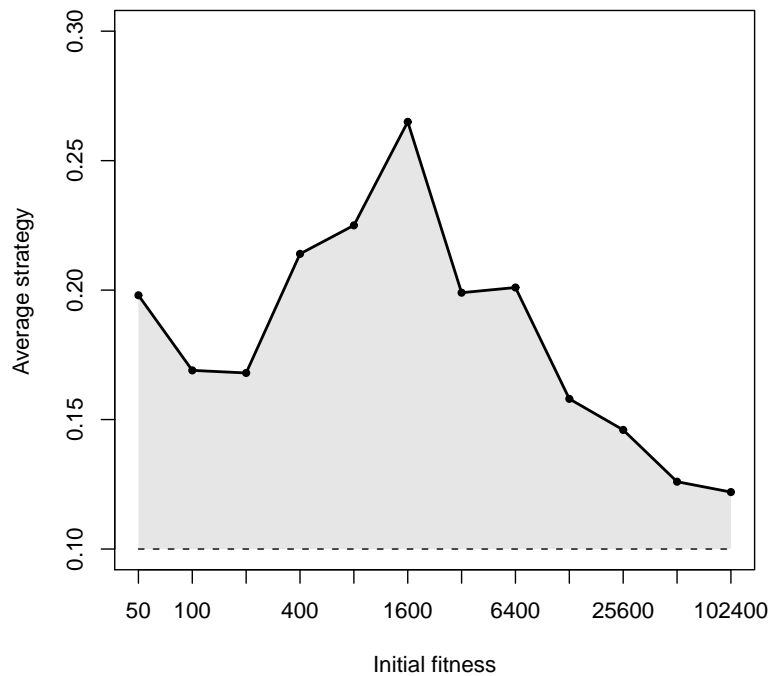

FIGURE 5.12 - Final average strategy of the population with different initial fitnesses.

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5.4.6 Extra gain

After analysing the effects of global satisfaction, represented as the initial fitness, we now analyse local satisfaction by working with extra gain. We give an extra gain of fitness to both agents at the end of each game. Only the agents that competed in a given turn receive it. The gain can be ecologically interpreted as a capacity of the environment to increase or decrease the fitness of the agents, from predators (negative gain) to preys (positive). Table 5.3 presents game C^g, which increases g to each payoff on the original chicken game. As we increase all the payoffs by a constant value, the expected payoff of each agent also increases with this value, and thus the equilibrium point does not change. Given that, both $C^{+0.0}$ and the chicken game shown in Table 5.1 represent the same game.

TABLE 5.3 - Payoffs of game with extra gain C^g, in pairs (A, B).

	B escalates	B does not escalate
A escalates	$(g - 10, g - 10)$	$(g + 1, g - 1)$
A does not escalate	$(g - 1, g + 1)$	(g, g)

We tested 17 models with extra gain, including the sequence $C^{-6.4}, C^{-3.2}, \ldots, C^{-0.2}$, $C^{-0.1}, C^{+0.0}, C^{+0.1}, C^{+0.2}, \ldots, C^{+3.2}, C^{+6.4}$, plus the extreme games $C^{-9.9}$ and $C^{+9.9}$. An extra gain below -9.9 would make the agents relocate in a compulsory way at least in each three turns, without any relation to local satisfaction, that should work as a metric for the composition of the other agents that belong to a cell. An extra gain above $+9.9$ makes the payoffs of the game always positive, and therefore nobody would ever relocate. It is important to note that, in these simulations, the population always start with only $s_{1.0}$.

Figure 5.13 shows the average strategy of the population after the model stabilises using different extra gains. Positive and negative extra gains have different results, which need to be analysed separately.

To understand the positive extra gain, first we need to compute what would happen with a non-spatial population, as in Maynard Smith's model. The expected payoff of a well-mixed population with average strategy s_x is $-10x^2$, if we do not consider any extra gain. Given that, the overall fitness of a population would not decrease if we give a gain above the expected payoff to its members after the games. It means that, given a population with average strategy s_x in a cellular space, their members would
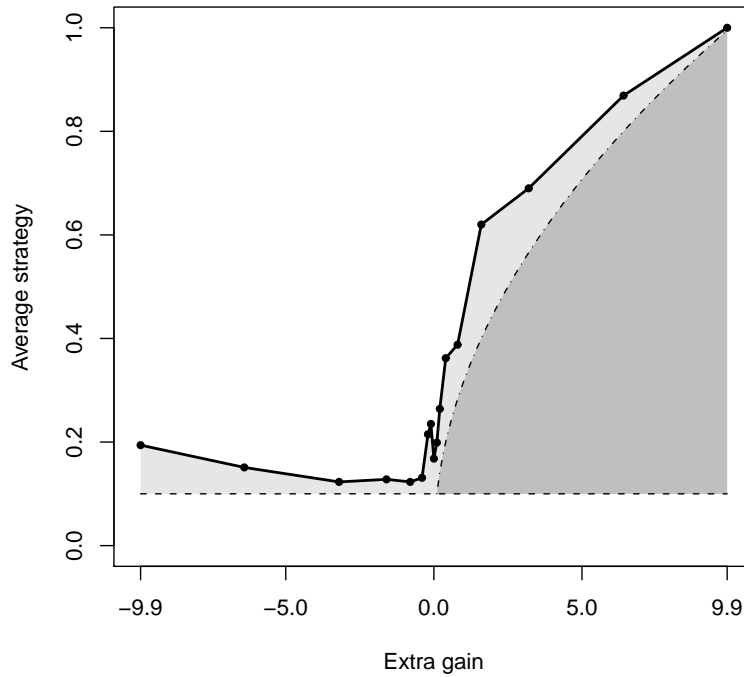
FIGURE 5.13 - Effects of extra gain on the evolution of the population.

not relocate if $x < \sqrt{\dfrac{g}{10}}$. The dark gray area in Figure 5.13 displays this situation.

Given that, simulations with positive extra gain never reached the 300th generation because they always found a stable point where the competition never finishes and no agent relocates. The populations only converged to those states with positive extra gain because they started with $s_{1.0}$. Otherwise, they could have stopped in the first generation. This way, the points of positive extra gain in Figure 5.13 represent the average strategy of the penultimate generation. It means that it is necessary to guarantee mobility in order to make the average strategy of the population get closer to the theoretical equilibrium point, otherwise the population would compete as a random drift within the dark gray area.

In the other side, negative extra gain indeed represents the average in the interval from the 100th to the 300th generation, as the population always converged to a stable state with at most one agent in each cell. The results show that the populations reach the points closest to the theoretical equilibrium when the gain is between $C^{-0.4}$ and $C^{-3.2}$. It means that, if the environment pressures the population with a considerable negative gain, it forces the population to get closer to the analytical

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equilibrium point. With lower gains ($C^{-6.4}$ and $C^{-9.9}$), the population approaches to the situation where the local satisfaction has no effect, as described previously, and the average strategy diverges from the theoretical equilibrium. Therefore, using local satisfaction effectively, allowing the movement of agents without making it a compulsory action, is important to maintain the population closer to the theoretical equilibrium point.

5.4.7 Games with different equilibrium points

In this subsection, we analyse the ability of the model to evolve with games that have different theoretical equilibrium points. To do so, let we introduce a game called C_e, which is a variation of the original chicken game. C_e applies e, instead of always 1, to the payoffs where one player escalates and the other does not, as shown in Table 5.4. The theoretical equilibrium point of game C_e is $s_{e/10}$. Within this context, both $C_{1.0}$ and the chicken game shown in Table 5.1 represent the same game.

TABLE 5.4 - Payoffs of game C_e, in pairs (A, B).

	B escalates	B does not escalate
A escalates	$(-10, -10)$	$(+e, -e)$
A does not escalate	$(-e, +e)$	$(0, 0)$

We simulated the evolution of populations using four games, $C_{2.0}$, $C_{4.0}$, $C_{6.0}$, and $C_{8.0}$, with equilibrium points of $s_{0.2}$, $s_{0.4}$, $s_{0.6}$, and $s_{0.8}$, respectively. The number of cells controlled by the different strategies at the end of each generation in all the four simulations is shown in Figure 5.14. We can see that, in each simulation, the equilibrium strategy ($s_{e/10}$) and the two right above ($s_{(e+1)/10}$ and $s_{(e+2)/10}$) are those that achieved most cells in the simulations. With the increase in the equilibrium point, the diversity of strategies in each generation also increases and never stabilises. However, even with different equilibrium points, the average strategy in the four simulations also converges to a stable state, close and above to the theoretical equilibrium points, as we can see in Figure 5.15.

Analysing the overall distribution of strategies after the model stabilises, now using four new simulations with mutation chance changing from 10% to 50% (because the strategies are more stable after convergence), we have the results shown in Figure 5.16. The distribution of individual strategies is smooth and follows a discretised
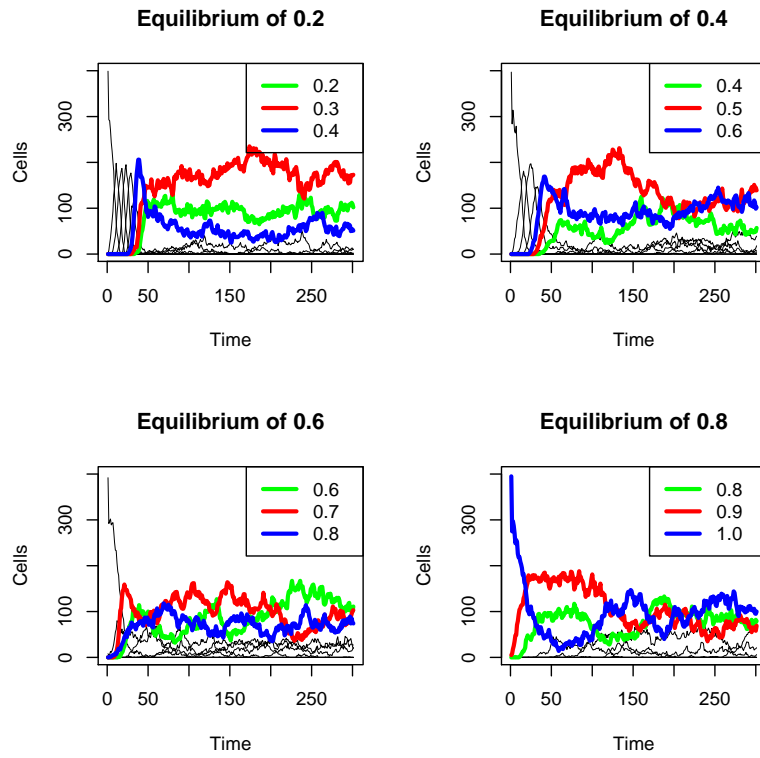
FIGURE 5.14 - Cells owned by different strategies in games with different equilibrium points.
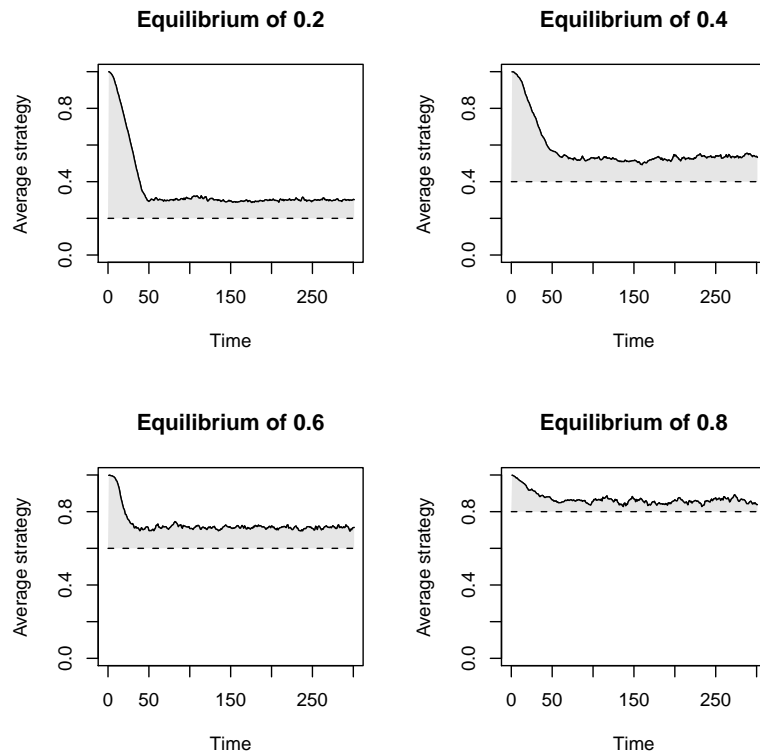

FIGURE 5.15 - Convergence of the average strategy in games with different equilibrium points.

gamma-like distribution, in the same way of the result shown in Figure 5.9 with mutation chance of 50%. The more successful strategy is always in the right side of the theoretical equilibrium point.
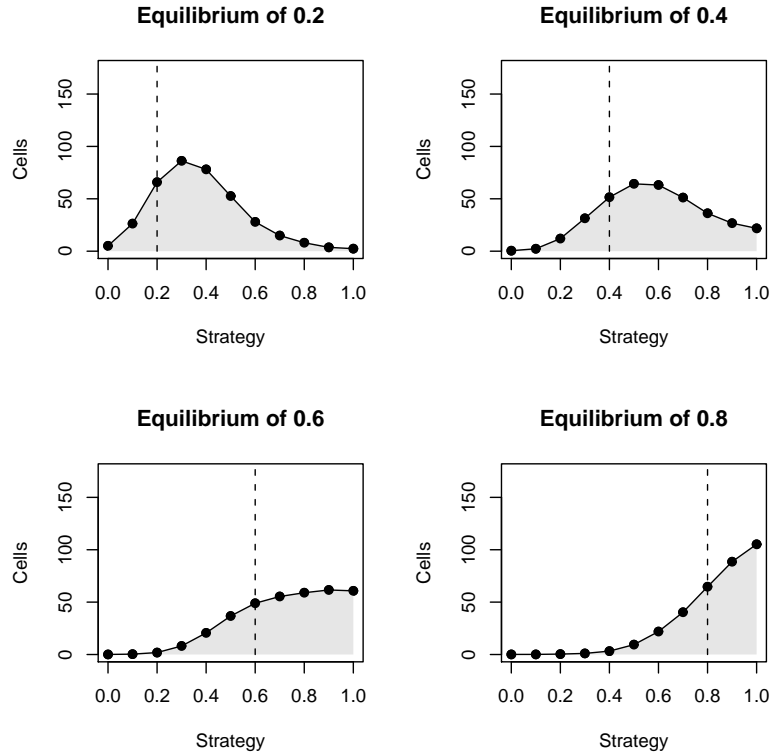

FIGURE 5.16 - Distribution of strategies in games with different equilibrium points, using 50% of mutation chance.

The results presented so far use games with equilibrium points from $s_{0.1}$ to $s_{0.8}$. None of the equilibrium points are close to pure strategies, that is, $s_{0.0}$ and $s_{1.0}$. Now we simulate the model with games whose equilibrium points get closer to these extreme points, simulating lower ($C_{0.2}$, $C_{0.4}$, $C_{0.6}$, $C_{0.8}$) and upper values ($C_{9.2}$, $C_{9.4}$, $C_{9.6}$, $C_{9.8}$).

Figures 5.17 and 5.18 show the average strategy of the population when the game gets closer to the pure strategy of never and always escalating, respectively. We can see that these results do not match the previous simulations. When the theoretical equilibrium point gets closer to $s_{0.0}$, the average also goes towards this point, but it changes faster than before, standing below the theoretical equilibrium point in three of the four simulations, in opposition of the previous results, where the pop-
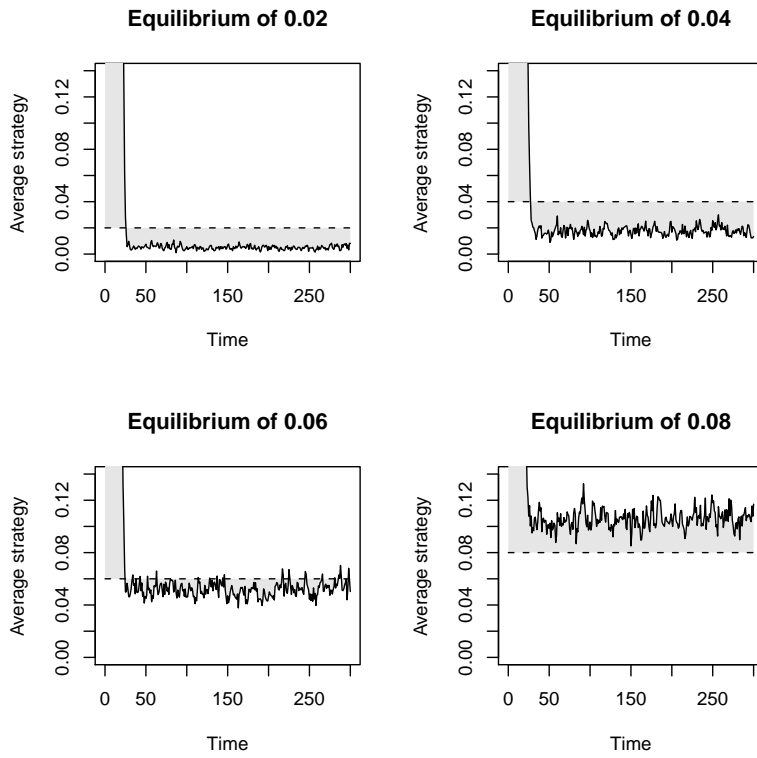
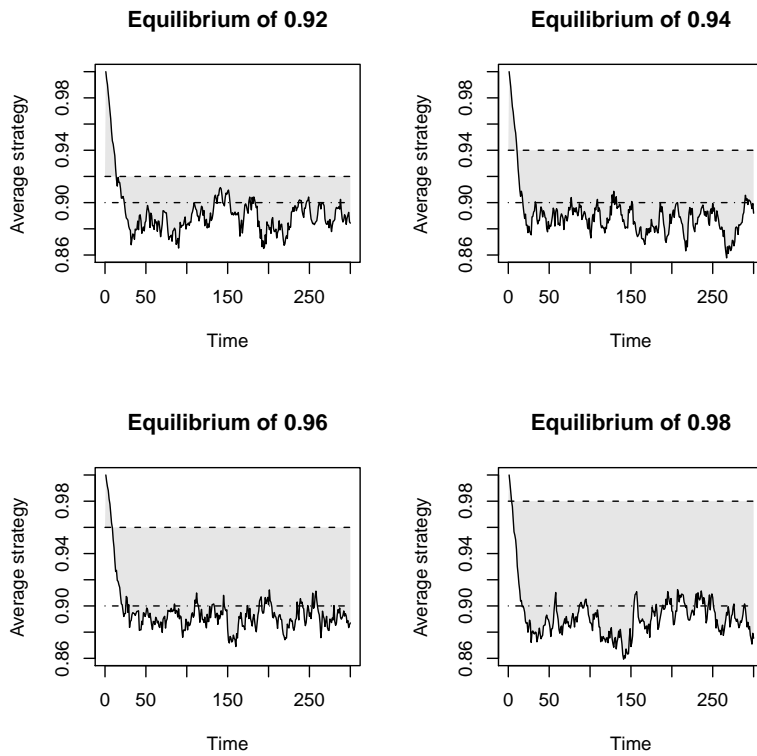FIGURE 5.17 - Theoretical equilibrium point getting closer to $s_{0.0}$.

FIGURE 5.18 - Theoretical equilibrium point getting closer to $s_{1.0}$.

ulation always stabilised above. The standard deviation of the average strategy is also proportional to the theoretical equilibrium.

On the other side, when the theoretical equilibrium point gets closer to $s_{1.0}$, neither the average nor the standard deviation change a bit. We can see that clearly due to the dotted-and-dashed lines showing $s_{0.9}$ in Figure 5.18; sometimes the simulations surpass it, but the averages are clearly below this point.

The results presented in the last two figures become clear if we plot the average strategy in the overall range of games simulated in this subsection. Figure 5.19 shows the final distribution of the averages, with the grey area representing one standard deviation around the average, with the dashed line showing the points where the average strategy would be equal to the analytical equilibrium point. The standard deviation is proportional to the equilibrium point from $s_{0.0}$ to $s_{0.6}$, where it reaches a maximum of 0.159. After that, it stabilises around 0.106. We can see that the theoretical equilibrium point always belongs to the interval of one standard deviation around the average strategy in the complete range of the studied equilibrium points.
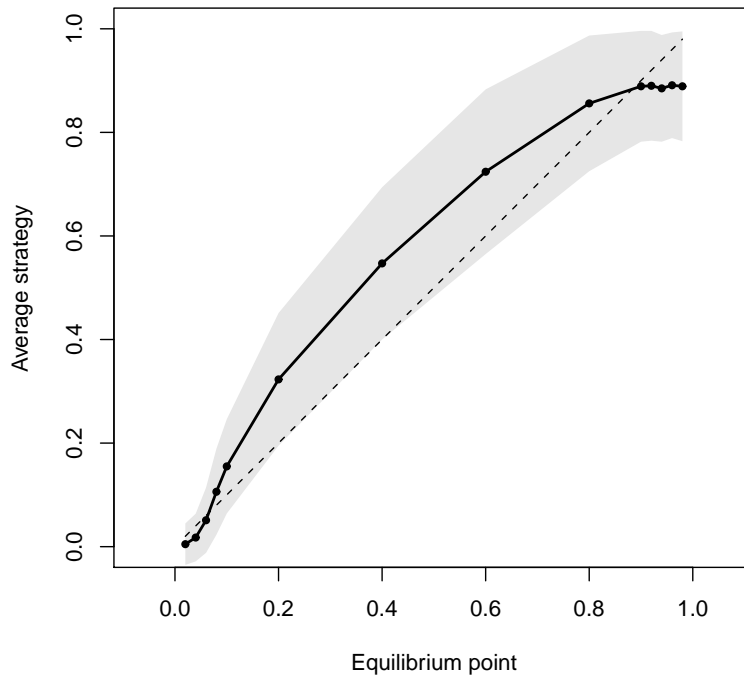

FIGURE 5.19 - Average strategy in the complete range of equilibrium points.

The curve of the simulation results crosses the dashed line twice. Executing a linear interpolation with the pairs of points whose interval crosses the line of theoretical equilibrium, we have $s_{0.065}$ and $s_{0.883}$ as the two strategies where the results of the simulations are equal to the theoretical equilibrium. However, these values are dependant on the configuration of parameter values used in the simulations.

The resulting curve is similar to a logistic equation, where the cooperative pure strategy (not to escalate) attracts and the more defective one repulses the average population. Therefore, populations are capable to adapt themselves even in games with different equilibrium points. The stable state of the population is usually above the theoretical equilibrium point, but it is affected by border effects towards cooperation. A conjecture from these results is that this logistic curve tends to get closer to the line of theoretical equilibrium points as the other parameters of the model move towards the values of the analytical model. Analysing this conjecture requires further investigation.

5.4.8   Adaptability

The last test of the model is related to the capability of the population to adapt itself to a change in the environment. We suppose that this kind of change affects the rules of the game, which can be represented by new values for the payoff matrix. If we change the game along the simulation, *will the population still converge to a stable state, close to the new equilibrium point?*

In this simulation, we started the population with only agents following $s_{1.0}$ and game $C_{2.0}$. As the populations of the previous results with similar configuration of parameters converge to a stable state in less than 50 generations, we change the game after each 100 of 300 generations. After the 100th generation, we change the payoffs in order to have an equilibrium point of $s_{0.6}$ (game $C_{6.0}$). Finally, after the 200th generation, we change the game again, this time to $C_{1.0}$.

The results of the simulations are shown in Figure 5.20. We can see that the population is capable to adapt itself to different games along the simulation. These results are similar to the ones shown before, once the individual strategies do not stabilise, but the average strategy always converges and stabilises close and above to the theoretical equilibrium point. It means that the model is capable to converge with whatever combination of strategies it starts. Moreover, heterogeneity plays a

central role in this case because, as more heterogeneous a population is, faster it can readapt itself to a new stable state.
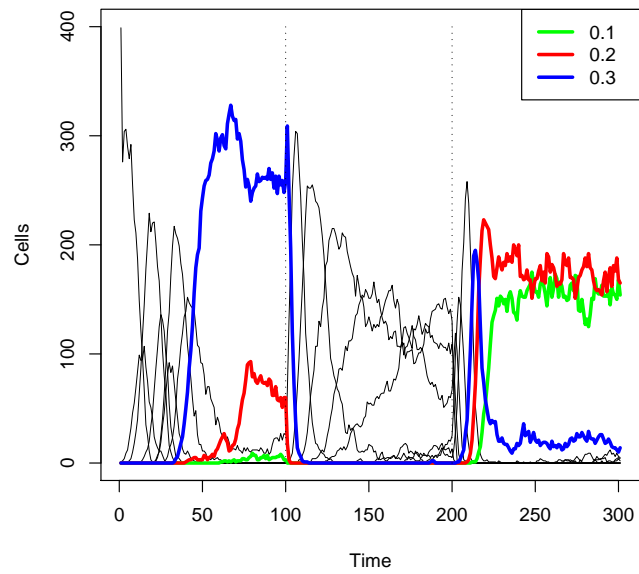
(a) Evolution of the different strategies.
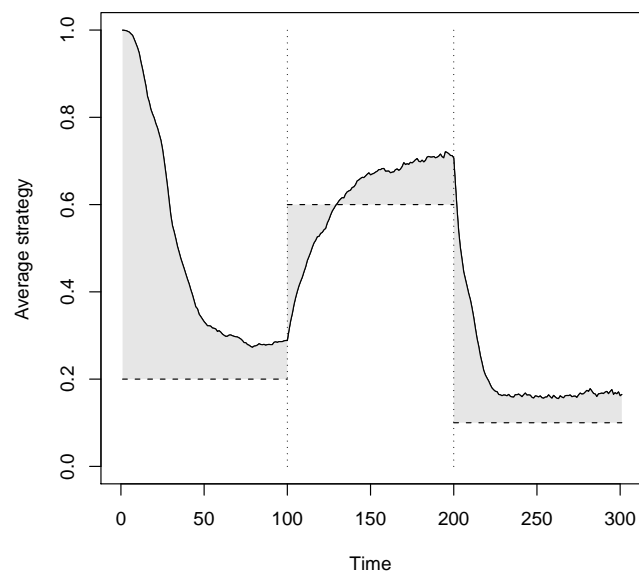
(b) Convergence of the average strategy.

FIGURE 5.20 - Adaptation to different games along the simulation.

5.4.9 Summary of the results

Table 5.5 summarises the effects of each parameter of the model, comparing the simulation results with the analytical values of Maynard Smith's model. Note that, in the parameters that there is some relation with the analytical model, the result tends to get closer the theoretical equilibrium point as the parameter goes towards the analytical value.

TABLE 5.5 - Summary of the effects of each parameter.

Parameter	Maynard Smith's model	Values used in the simulations	Effect on the stable state when changing the parameter
Initial strategy	Not applicable.	$s_{0.0}$, $s_{0.1}$, $s_{0.5}$, and $s_{1.0}$.	No effect.
Initial fitness	Infinite, as resources are not limited.	50, 100, 200, 400, 800, 1600, 3200, 6400, 12800, 25600, 51200, and 102400.	Distance to the theoretical point inversely proportional to the initial fitness.
Extra gain ($\mathbf{C}^g$)	Not applicable.	$C^{-9.9}$, $C^{-6.4}$, $C^{-3.2}$, $C^{-1.6}$, $C^{-0.8}$, $C^{-0.4}$, $C^{-0.2}$, $C^{-0.1}$, $C^{+0.0}$, $C^{+0.1}$, $C^{+0.2}$, $C^{+0.4}$, $C^{+0.8}$, $C^{+1.6}$, $C^{+3.2}$, $C^{+6.4}$, and $C^{+9.9}$.	Distance to the theoretical point proportional to positive extra gain. Inversely proportional to negative gain.
Mutation probability	Zero, although it is considered that a mutation may emerge.	0.1%, 0.5%, 1%, 2%, 5%, 10%, 20%, and 50%.	Distance to the theoretical point proportional to the mutation probability.
Mutation change	Zero, although it is considered that a mutation may emerge.	±0.0005, ±0.001, ±0.005, ±0.01, ±0.05, and ±0.1.	Distance to the theoretical point proportional to the mutation change.
Equilibrium point ($\mathbf{C}_e$)	Not applicable.	$C_{0.2}$, $C_{0.4}$, $C_{0.6}$, $C_{0.8}$, $C_{1.0}$, $C_{2.0}$, $C_{4.0}$, $C_{6.0}$, $C_{8.0}$, $C_{9.0}$, $C_{9.2}$, $C_{9.4}$, $C_{9.6}$, and $C_{9.8}$.	Logistic curve in the range of theoretical points, with usual stabilisation above the theoretical equilibrium point. Cooperation works as an attractor and defection as a repulser.

5.5 Conclusions

By relaxing more assumptions than other works in the literature, adding a representation of space, limited resources, and a definition of mobility, we have a model that is capable to converge to a stable state with any initial configuration of its population. The results of our experiments show that, although the agents do not have any knowledge about equilibrium points and cannot even change their individual strategies along their lifetime, the population evolves and organises itself in a stable state, in spite of the fact that the frequencies of each strategy never stabilise along the simulations. The results also agree with the statements of Nash, who said that "we can only expect some sort of approximate equilibrium, since [...] the stability of the average frequencies will be imperfect" (NASH, 1950). As it is almost impossible to validate this kind of model with real world data, showing that the model is independent of the initial population is a reasonable way to make it more trustworthy.

Generally, when we add more complexity to a simulation model, it tends to move towards an unpredicted behaviour, but it is not what happens in the model presented in this paper. It means that space is a very important dimension in the evolution of populations because it limits the population size and define how the agents will interact with each other. Also, adding a genetic component makes the model change slowly, in opposition to the literature, where the population can change drastically from one generation to another. Indeed, evolution is a slow but continuous process, which occurs in changes that affect one single or a small group of agents, spreading over the population if it represents an evolutionary advantage. The model presented in this paper replicates this general behaviour.

The results of the model also points out how heterogeneity is important in population compositions, mainly because of the adaptation necessary to react to changes in the environment. In terms of evolution, it is more important to abdicate of a stable population in a complete equilibrium in order to have more chance to adapt faster to changes in the environment.

The simulation results corroborate the hypothesis of Maynard Smith, that populations are capable to evolve to stable states, differently from the results of the authors that relax other assumptions but pairwise contests and genetic representation (ORZACK; HINES, 2005; FOGEL *et al.*, 1997; FOGEL *et al.*, 1998). However, the

conclusions from the results of the simulations have shown some disagreement if compared to Maynard Smith's model, which are related to the complexity added to the model. They are:

a) We conjecture that, in the case where there is a mixed equilibrium point, a population is in equilibrium when the *average strategy* is in equilibrium. It includes as special cases both solutions presented by Maynard Smith, that is, a population where every agent follows the same mixed strategy, or where there is a fixed proportion of two pure strategies.

b) The stable state is approximately the theoretical equilibrium point. In the simulations, it is usually above the theoretical point, but when the theoretical mixed equilibrium point is close to a pure strategy, cooperation becomes an attractor and defection a repulser. The fact that the population does not attain the theoretical equilibrium point is not a simple misconfiguration of the parameters of the model, as we have shown with a reasonable number of simulations changing each parameter separately. Our conjecture is that, when the parameters of the model tend to the values of the theoretical model (fitness towards infinity, mutation rate and probability tending to zero), the average strategy converges to the theoretical equilibrium point.

c) Mobility plays a central role to guarantee that a population converges to a stable state closer to the theoretical equilibrium point. Otherwise the population would compete as a random drift, or even diverge from the theoretical equilibrium if the mobility is independent of the results of the game.

Finally, the results of the model here presented could lead to a wrong game theoretical analysis. The traditional literature of Game Theory recommends that, given a real situation where a population is supposed to be in equilibrium, the difficulty relies only on estimating the game payoffs to show that the phenomenon is indeed in equilibrium. However, when the payoffs of the game are connected to other factors that may affect the equilibrium of the population, as shown in this work, it is necessary to remove these effects in order to find out the real payoffs. And here comes the difficulty, because it is completely possible to estimate wrong payoffs from the simulation results without removing these side effects, mainly because the population indeed reaches a stable state. But this induced game may not be the one that

the population is playing, leading to a wrong understanding of the phenomenon. For example, if a given population has reached an average strategy of $s_{0.195}$ from a game with equilibrium point of $s_{0.1}$, a game-theoretical analysis could infer that the players interact with each other through the game $C_{0.195}$, ignoring all other factors that can affect this equilibrium state. Using an agent-based approach is a good way to try to overcome this hurdle.

6 FINAL REMARKS

This thesis presented a couple of advances related to the development and exploration of spatially explicit agent-based models. Its main contributions are:

a) A general architecture for dealing with geospatial phenomena in an agent-based approach;

b) A new concept of games on cellular spaces, with a clear separation between the definitions of cell and agent.

c) The observation that, within a spatial dynamics, there are more factors than just the theoretical equilibrium point influencing the stability of a population;

d) A generalisation of the interpretation of Maynard Smith on the meaning of a stable state of a population.

Additionally, there are some scientific advances related to this thesis currently being developed or planned to be investigated by the author and other researchers. Some of them worth to be cited:

Game theory. The best formal approach for modelling conflict is Game Theory. Some works can be cited for future directions in the line of games on cellular spaces, putting forward the results presented in Chapters 4 and 5. The main question that remains unanswered is the conjecture about how the shape of the logistic curve would behave when the parameters of the model get closer to the values of the analytical model. Additionally, we could study games with incomplete information, coalitions, or relax the assumption of non-overlapping generations. A more applied direction of this line can be by working with models of common pool resources management and institutional arrangements, following the studies of Elinor Ostrom (OSTROM, 1990; OSTROM *et al.*, 1994).

Economic chains. This work examines economic scenarios and its impacts on economic chains related to strategies of payment for carbon credits in the Southeast Pará state. It studies a well established economic methodology (input-output analysis), investigating its implicit assumptions within an

agent-based perspective[1]. The next steps of this specific work is to add a spatially explicit representation to the agents, grounding them to a representation of the real world. Clearly, the architecture presented on Chapter 3 is the way for establishing these relations.

Relations and land change models. This work applies the relations presented in Chapter 3 to study land use change processes related to a formal agreement, the soybean moratorium pact, in Santarém, Pará[2]. This is part of the Master's degree work of Talita Assis, that studies the effects of institutional arrangements in the deforestation processes within Amazonia.

Temporal relations. Chapter 3 focus on a static representation of geospatial data to feed models. The work on temporal relations consists currently on developing a typology for modelling situations where the neighbourhood structure is capable to change along the simulation, using or not using exogenous geospatial data. It is part of the Master's degree work of Raian Maretto[3].

TerraME Project. TerraME[4] is a development environment for spatial dynamical modelling that links cellular spaces to geospatial databases for data storage and retrieval. It was initially developed as a Doctorate Thesis (CARNEIRO, 2006), and now it is a joint project between INPE and Federal University of Ouro Preto (UFOP). New methodologies have been proposed to increase its functionalities, such as (LIMA *et al.*, 2008; MOREIRA *et al.*, 2008). The architecture proposed in Chapter 3 has been implemented in TerraME, in a long effort to define general types to satisfy all spatially explicit dynamic models for studying human-environment interactions[5]. Finally, advances on the previous research lines almost always rely on further development of TerraME.

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[1]P. R. Andrade, A. M. V. Monteiro, G. Camara, *From Input-Output Matrixes to Agent-Based Models: A Case Study on Carbon Credits in a Local Economy.* Second Brazilian Workshop of Social Simulation, BWSS'10.

[2]T. O. Assis, A. P. D. Aguiar, G. Camara, P. R. Andrade, *Modelos Baseados em Agentes em Mudanças de Uso e Cobertura da Terra: o Caso da Moratória da Soja em Santarém.* XI Brazilian Symposium on GeoInformatics (in Portuguese).

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