Center for Spatially Integrated Social Science

Spatial Autocorrelation (2) Spatial Weights

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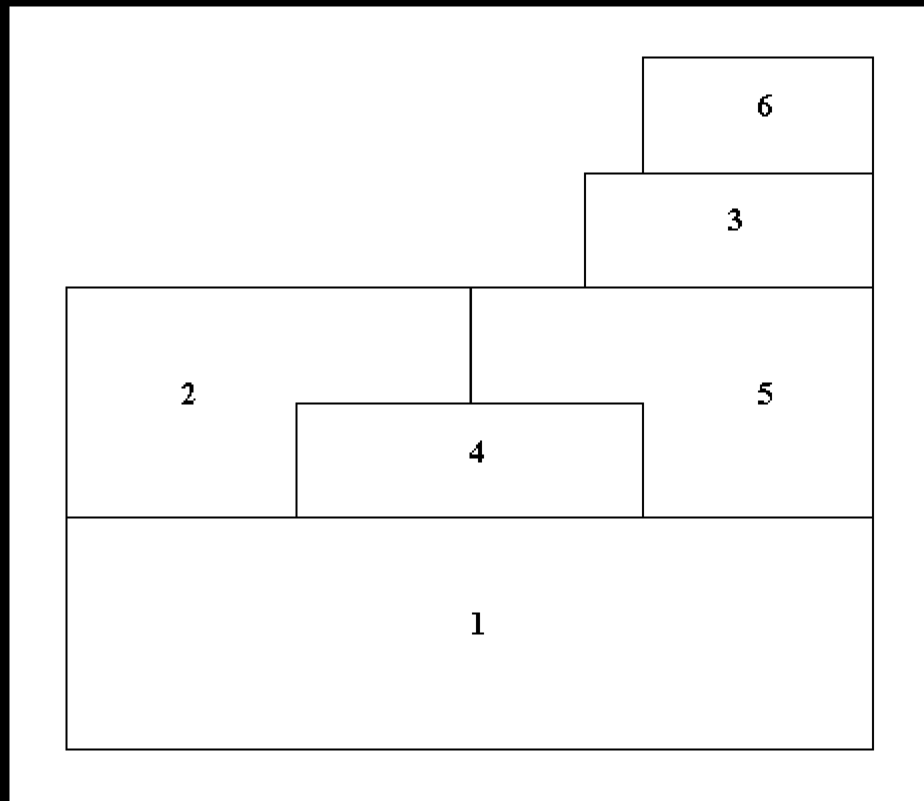
Outline

- Concepts
- Contiguity Weights
- Other Spatial Weights
- Spatial Lag Operator

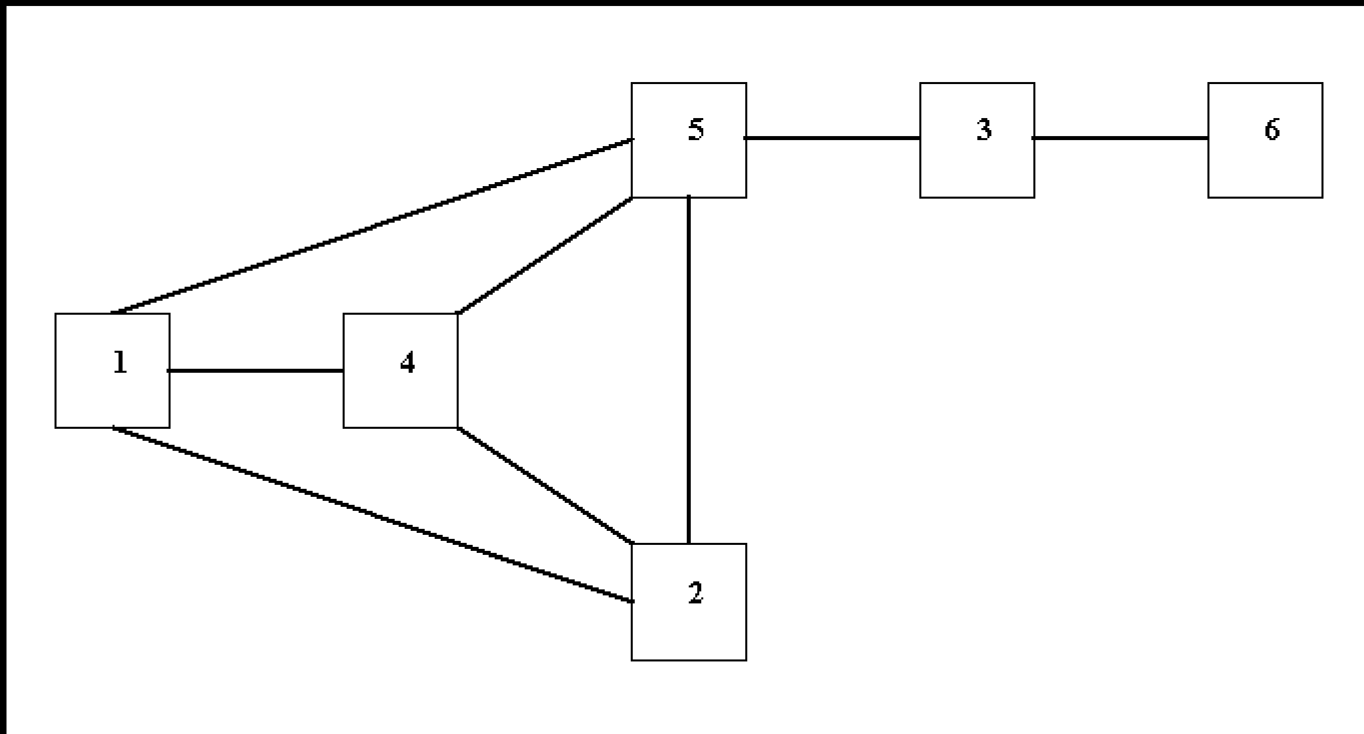
Concepts

Why Spatial Weights?

- Spatial Correlation
  - $Cov[y_i . y_h] \neq 0$, for $i \neq h$
- Structure of Correlation
  - which i, h interact?
  - N observations to estimate N(N-1)/2 interactions
  - impose structure in terms of what are the "neighbors" for each location



Example: N=6
contiguity = common boundary



contiguity as a graph
link between nodes = contiguity

0	1	0	1	1	0
1	0	0	1	1	0
0	0	0	0	1	1
1	1	0	0	1	0
1	1	1	1	0	0
0	0	1	0	0	0

First Order Contiguity Matrix

Spatial Weights Matrix

- Definition
  - N by N positive matrix W, with elements w_{ij}
- Simplest Form: Binary Contiguity
  - $w_{ij} = 1$ for i and j "neighbors" (e.g. $d_{ij} <$ critical distance)
  - $w_{ij} = 0$ otherwise, $w_{ii} = 0$ by convention
- Row Standardization
  - averaging of neighboring values
  - $w^s_{ij} = w_{ij} / \Sigma_j w_{ij}$ such that $\Sigma_j w^s_{ij} = 1$
  - spatial parameters comparable

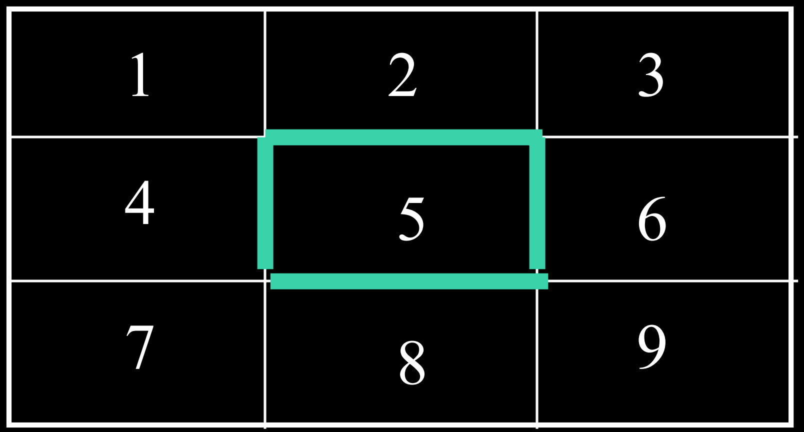
How to Construct Weights

- Contiguity
  - common boundary
- Distance
  - distance band
  - k-nearest neighbors
- General
  - social distance
  - complex distance decay functions

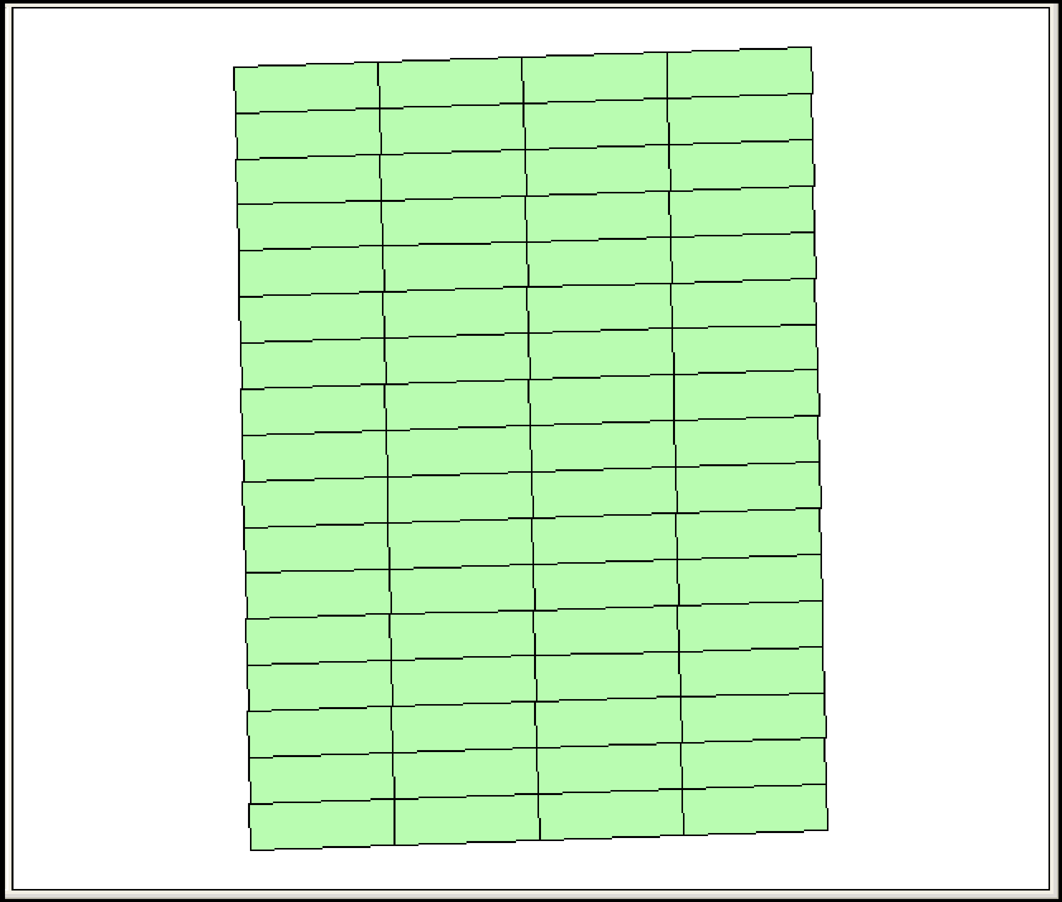
Contiguity Weights

Contiguity – Regular Grid

- Regular Grid
  - rook
    - 2, 4, 6, 8
  - bishop
    - 1, 3, 7, 9
  - queen
    - both

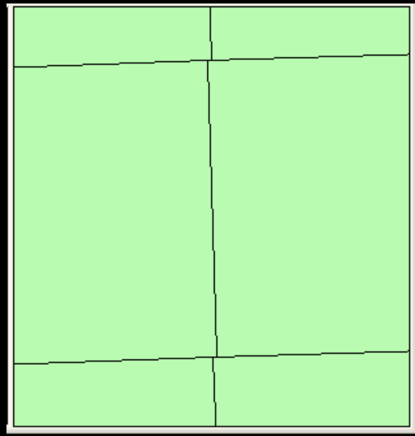


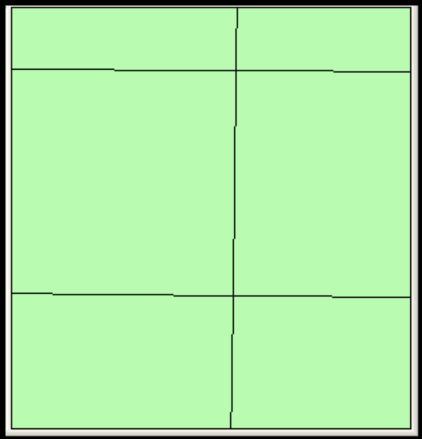
Regular Grid Polygons



Contiguity and GIS Precision

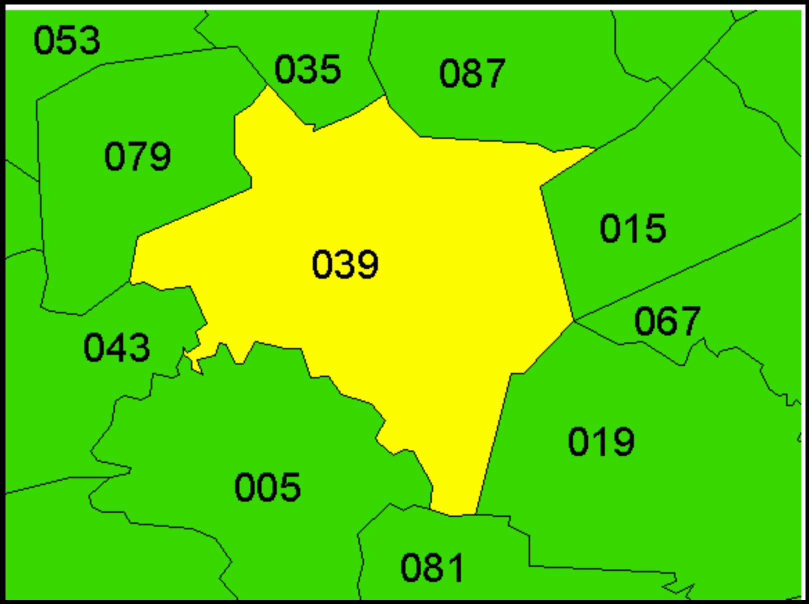
- The regular “grid” in the top picture is actually misaligned at the corners
- The grid on the bottom picture has the proper contiguity



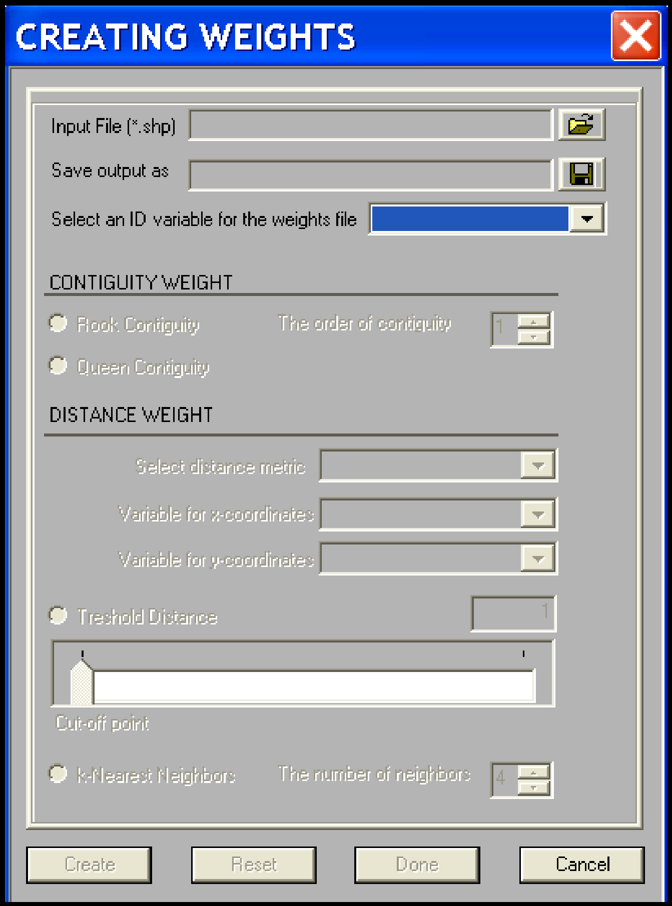


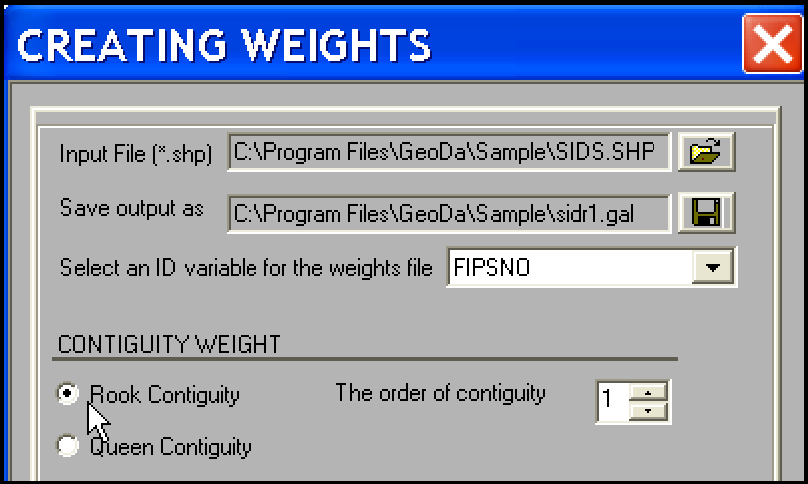
Contiguity – Irregular Units

- Irregular Units
  - common border
    - rook
  - common vertex
    - 039 and 067
    - queen



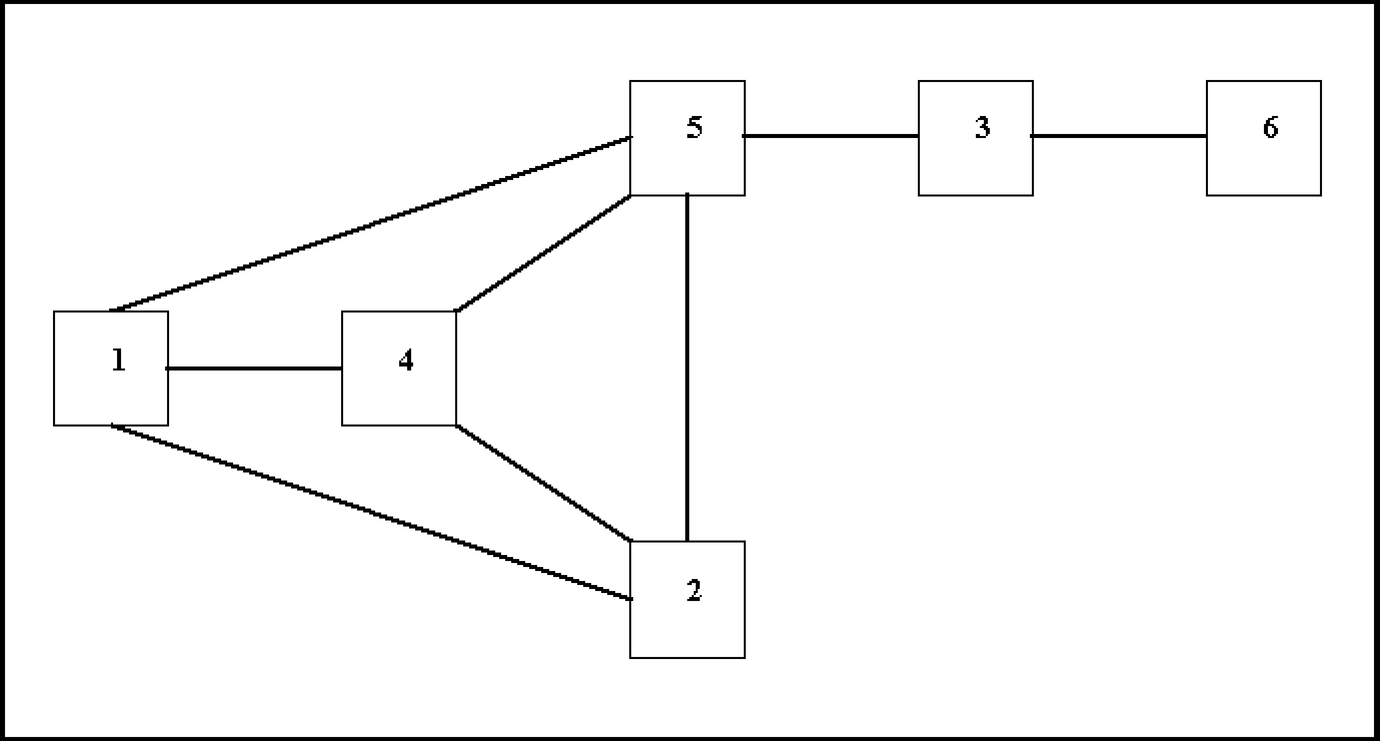
Contiguity Weights in GeoDa





Higher Order Contiguity

- Recursive Definition
  - j contiguous to i of order p:
    j first order contiguous to k
    k contiguous to i of order p-1
  - i and j not already contiguous of lower order
- Spatial Weights Matrix
  - representation of network or graph



contiguity as a graph
link between nodes = contiguity

Circularity and Redundancy in Higher Order Weights

- Powering of the Weights Matrix
  - standard approach invalid for contiguity
- Removing Circularity and Redundancy
  - sparse network representation of weights
  - modified Dijkstra algorithm to identify number of steps between nearest neighbors (Anselin and Smirnov)
  - number of steps = order of contiguity

0	1	0	1	1	0
1	0	0	1	1	0
0	0	0	0	1	1
1	1	0	0	1	0
1	1	1	1	0	0
0	0	1	0	0	0

First Order Contiguity Matrix

3	2	1	2	2	0
2	3	1	2	2	0
1	1	2	1	0	0
2	2	1	3	2	0
2	2	0	2	4	1
0	0	0	0	1	1

Second Power of First Order Contiguity

0	0	1	0	0	0
0	0	1	0	0	0
1	1	0	1	0	0
0	0	1	0	0	0
0	0	0	0	0	1
0	0	0	0	1	0

Correct Second Order Contiguity Matrix

-1	1	2	1	1	3
1	-1	2	1	1	3
2	2	-1	2	1	1
1	1	2	-1	1	3
1	1	1	1	-1	2
3	3	1	3	2	-1

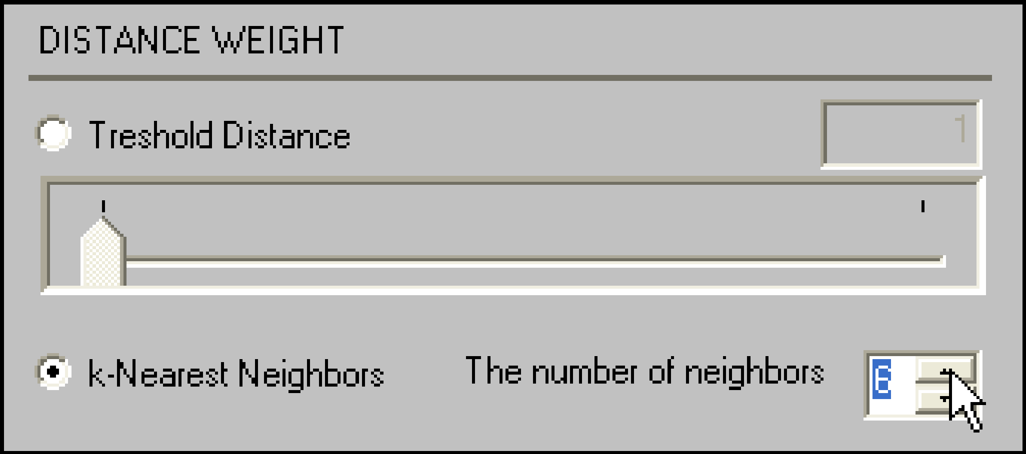
Matrix Constructed by Bottom-Up Algorithm

Other Spatial Weights

Distance-Based Weights

- Distance Band
  - $w_{ij} = 1$ for $d_{ij} <$ distance cut-off
- K-Nearest Neighbor Weights
  - k neighbors, irrespective of actual distance
  - "warps" space

Distance Based Weights



General Spatial Weights

- Cliff-Ord Weights
  - w_{ij} to reflect potential spatial interaction between i and j
  - $w_{ij} = [d_{ij}]^{-a}.[b_{ij}]^b$
    - with
      d_{ij} as distance between i and j
      b_{ij} as share of common boundary between i and j in perimeter of i

General Spatial Weights
(continued)

- Weights May Contain Parameters
  - inverse distance weights
    - $w_{ij} = 1 / d_{ij}^{\alpha}$
  - estimated from data or chosen a priori
    - in practice: second power (gravity model)
  - identification problems in nonlinear weights
    - interaction is multiplicative: $\rho \,.\, w_{ij} = \rho \, (1 / d_{ij}^{\alpha})$
    - parameters ρ and α not separately identified

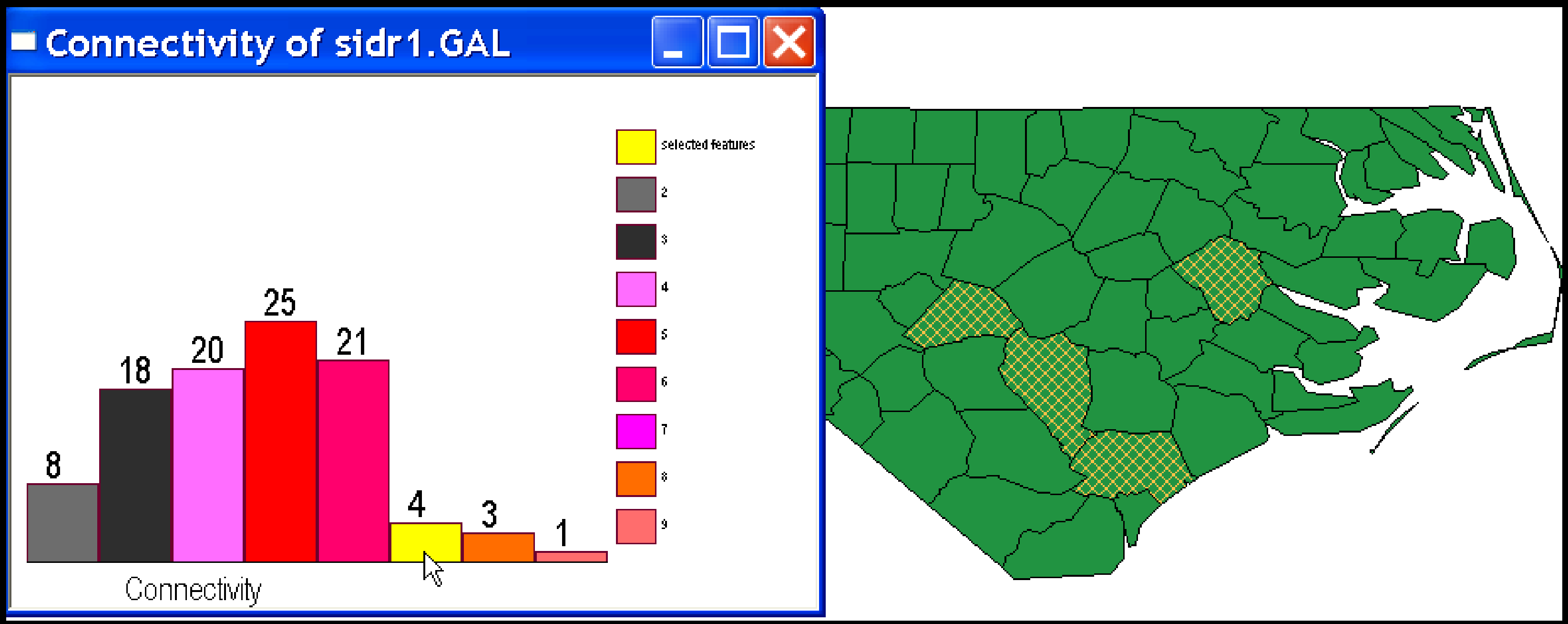
General Spatial Weights
(continued)

- Economic Weights (Case)
  - block structure, state effect
    - $w_{ij} = 1$ for all i, j in "block"
  - economic distance $|r_i - r_j|$
  - economic weight = $1/|r_i - r_j|$
    - e.g., r = total employment

Characteristics of Spatial Weights

- Measures of Overall Connectedness
  - percent nonzero weights (sparseness)
  - average weight
  - average number of links
  - principal eigenvalue
- Location-Specific Measures
  - most/least connected observations
  - unconnected observations = islands

NC Counties with 4 Neighbors



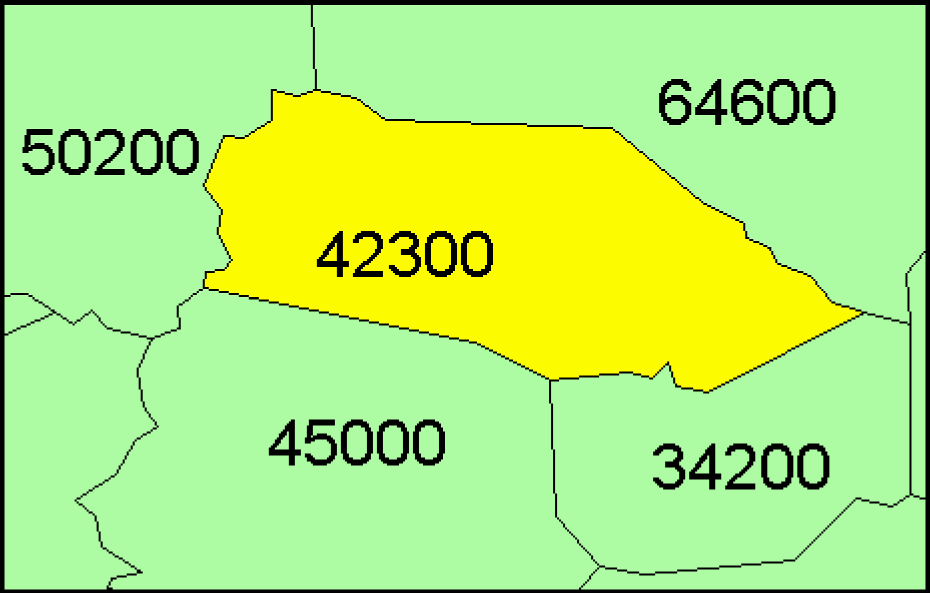
Spatial Lag Operator

Spatial Shift

- No Direct Counterpart to Time Series Shift Operator
  - time series: $L_k = y_{t-k}$
  - spatial series: which h are shifted by "k" from location i?
    - on regular lattice: east, west, north, south
    - (i - 1, j) (i + 1, j) (i, j - 1) (i, j + 1)
  - arbitrary for irregular lattice
    - different number of neighbors by observation

Spatial Lag Operator

- Distributed Lag
  - row-standardized weights $\Sigma_j w_{ij} = 1$
  - spatial lag is weighted average of neighboring values
    - $\Sigma_j w_{ij}.y_j$, for each i
    - vector Wy
    - spatial lag does not contain y_i
  - spatial lag is a smoother
    - not a window average



value y_i = $42,300

4 neighbors

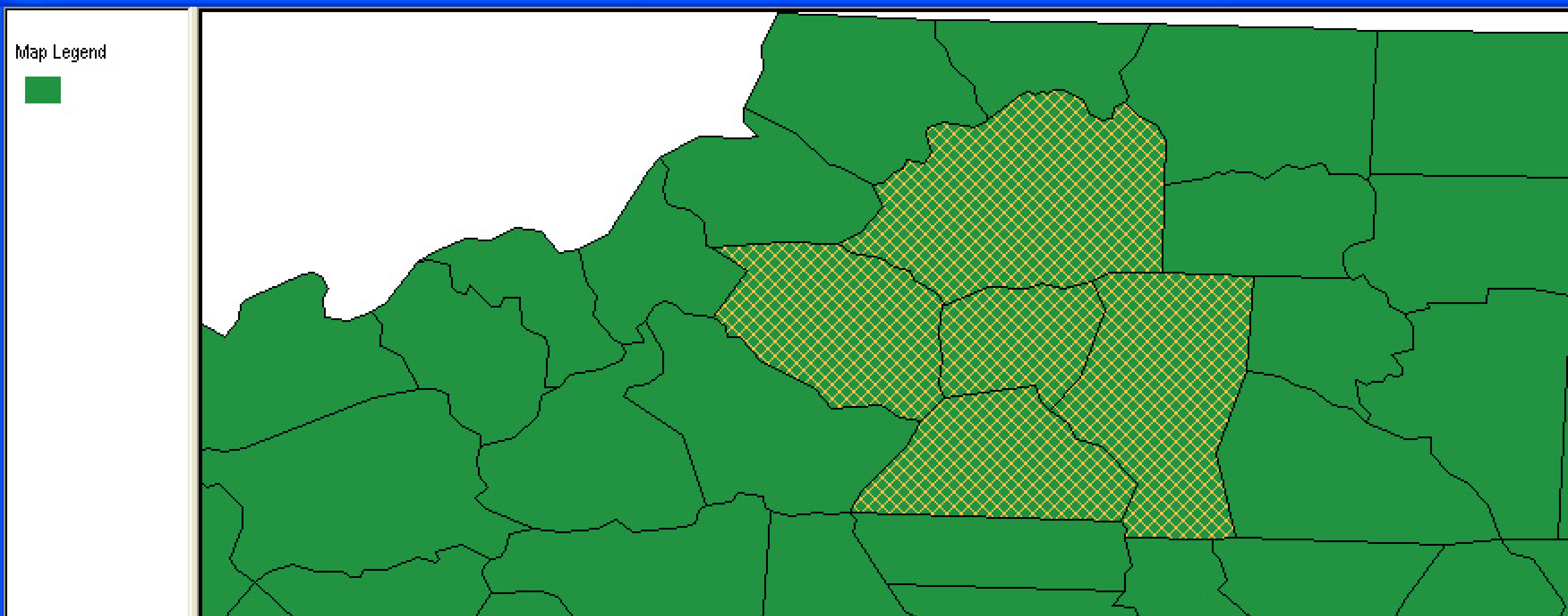
values for neighbors: $50,200, $64,600, $45,000, $34,200

spatial lag = (1/4)$50,200 + (1/4)$64,600 + (1/4)$45,000 + (1/4)$34,200 = $48,500

Table : SIDS

	CNTY_	CNTY_ID	NAME	STATE_NAME	STATE_FIPS	CNTY_FIPS	FIPS
52	1986	1986	Catawba	North Carolina	37	035	37035
39	1947	1947	Iredell	North Carolina	37	097	37097
41	1950	1950	Alexander	North Carolina	37	003	37003
34	1932	1932	Caldwell	North Carolina	37	027	37027
18	1874	1874	Wilkes	North Carolina	37	193	37193

SIDS



Spatial Lags in GeoDa Table

SIDR74	SIDR79	NWR74	NWR79	W_SIDR74
0.868961	3.050995	137.295794	132.790934	1.527684
0.966417	0.925926	276.395265	241.666667	1.198714
0.000000	1.188354	96.024006	89.126560	1.192336
1.662510	2.118145	85.619285	84.725818	0.715943
1.271456	1.879195	63.572791	59.597315	0.832222
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

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