Instituto Superior de Estatística e Gestão de Informação
Universidade Nova de Lisboa

Master of Science in Geospatial Technologies

Geostatistics Predictions with Anisotropy and Simulations

Carlos Alberto Felgueiras
cfelgueiras@isegi.unl.pt

Predictions with Anisotropy and Simulations

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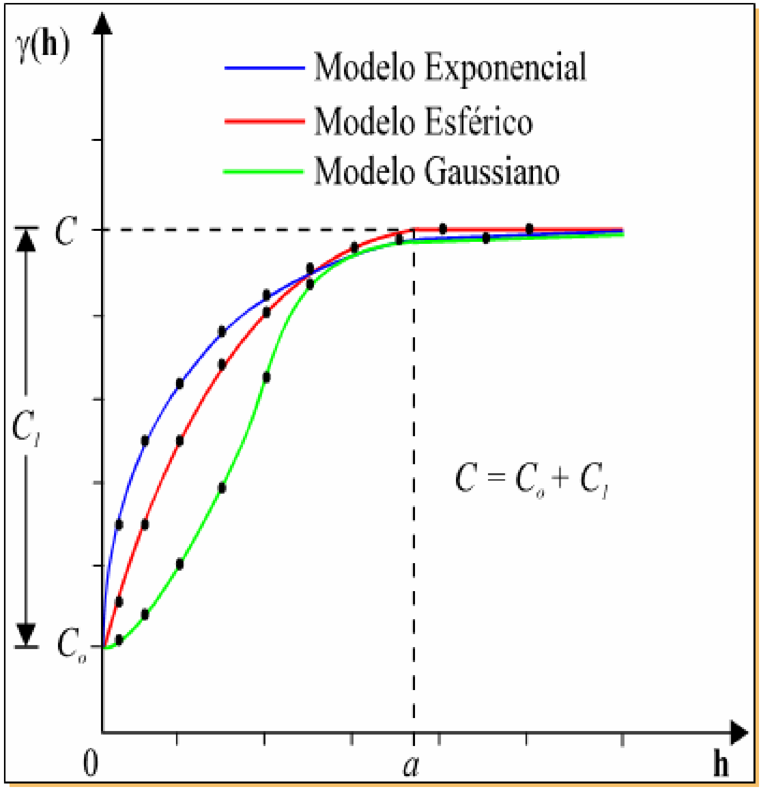
Stochastic Predictions with anysotropy

Predictions with Anisotropy and Simulations

- **Introduction**
  - **Unidirectional Semivariograms – Fitting with only one model**

Represent spatial variability of the attribute in one specific direction



Experimental Semivariogram
(from samples)

$$\gamma(\mathbf{h}) = \frac{1}{2N(\mathbf{h})} \sum_{i=1}^{N(\mathbf{h})} [\, z(\mathbf{u}_i) - z(\mathbf{u}_i + \mathbf{h})]^2$$

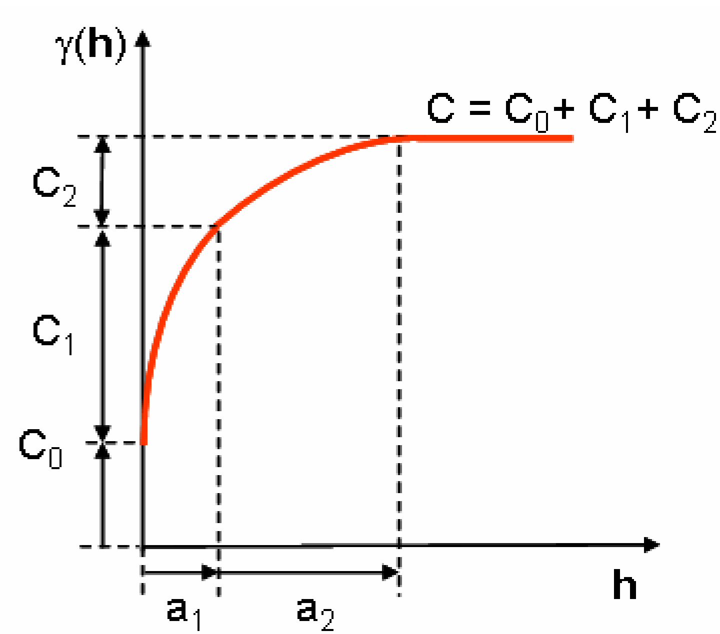
Theorical (Modeled) Semivariogram
(fitted from the experimental semivariogram using only one model)

$$\gamma(\mathbf{h}) = C_0 + C_1 \cdot \mathrm{Exp}\left(\frac{\mathbf{h}}{a}\right)$$

$$= C_0 + C_1 \cdot \left[1 - e^{\left(-\frac{|\mathbf{h}|}{a}\right)}\right]$$

Predictions with Anisotropy and Simulations

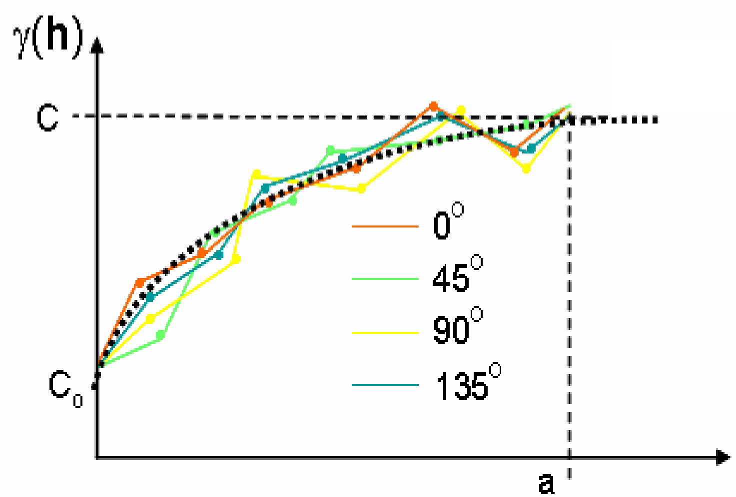
- **Introduction**
  - **Unidirectional Semivariograms – Fitting with Nested Models**

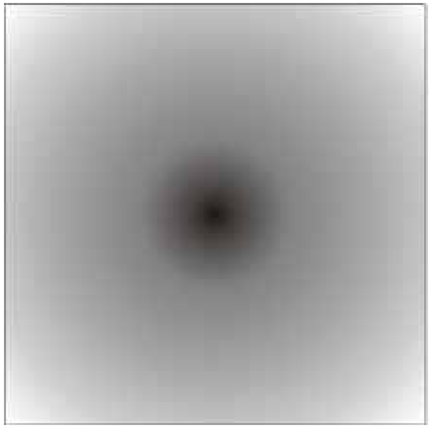


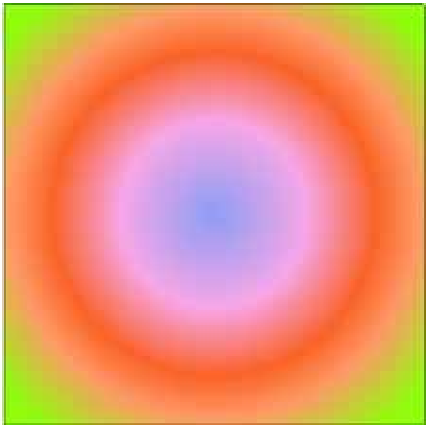
$$
\gamma(\mathbf{h}) = \begin{cases} 0, & C_0 \\ C_0 + C_1\left[\frac{3}{2}\left(\frac{|\mathbf{h}|}{a_1}\right) - \frac{1}{2}\left(\frac{|\mathbf{h}|}{a_1}\right)^3\right] = \gamma_1(\mathbf{h}), & 0 < |\mathbf{h}| \le a_1 \\ C_0 + C_2\left[\frac{3}{2}\left(\frac{|\mathbf{h}|}{a_2}\right) - \frac{1}{2}\left(\frac{|\mathbf{h}|}{a_2}\right)^3\right] = \gamma_2(\mathbf{h}), & a_1 < |\mathbf{h}| \le a_2 \\ C_0 + C_1 + C_2, & |\mathbf{h}| > a_2 \end{cases}
$$

Predictions with Anisotropy and Simulations

- **Isotropy x Anysotropy**
- **Isotropic Spatial Variation - Omnidirectional Semivariogram**
  - Defined by:
    - Any Angular Direction (0 degrees for example)
    - Angular Tolerance equal 90 degrees for up and down directions (completing 360 degrees. Why?)





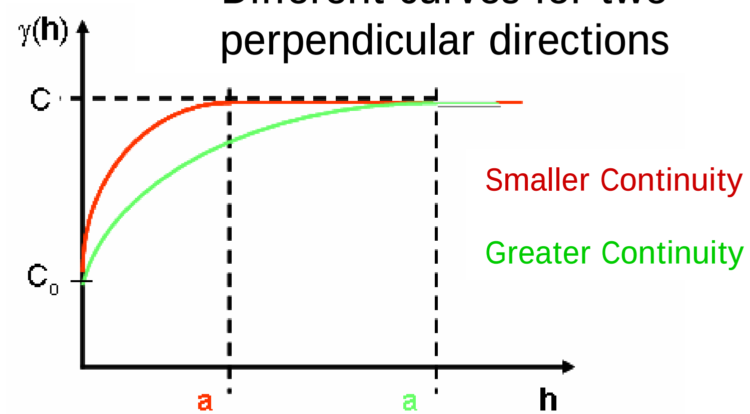


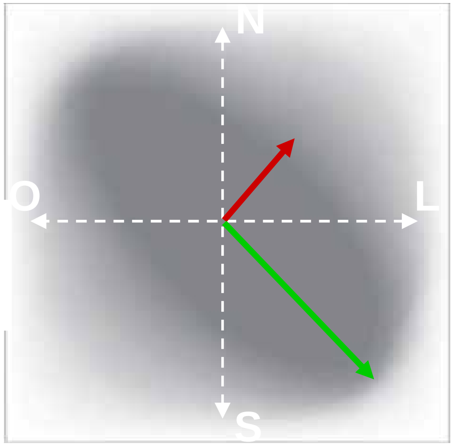
Semivariogram for 4 different directions and semivariogram surfaces

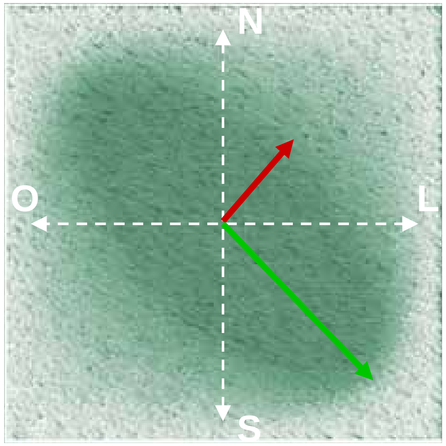
Predictions with Anisotropy and Simulations

- **Isotropy x Anisotropy**
- **Anisotropic Spatial Variation – 2 Directional Semivariograms**
  - Defined by:
    - Angular Directions of the greatest and the smallest spatial continuity
    - Angular Tolerance much lesser than 90 degrees for up and down directions (30 degrees for example can be the first try)

Different curves for two perpendicular directions



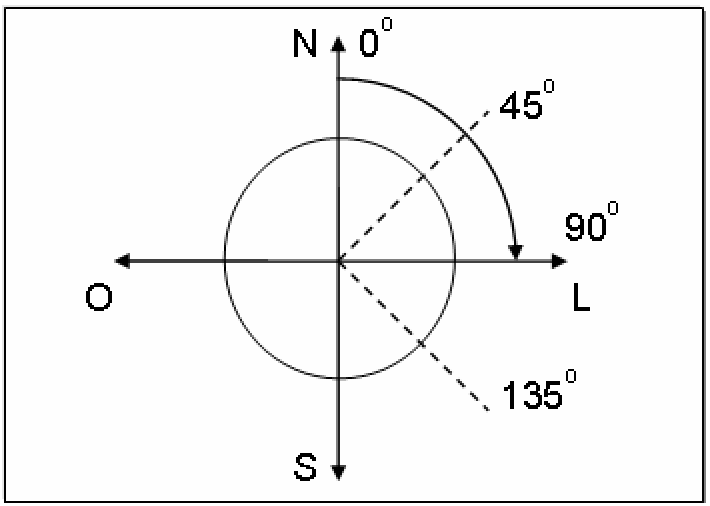


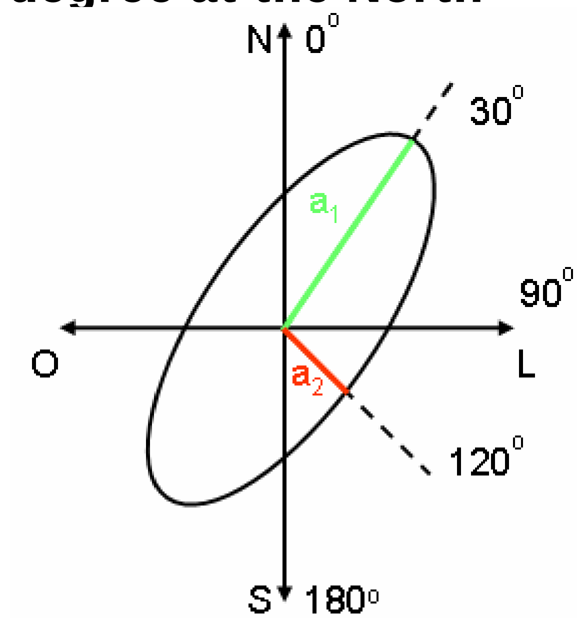


Semivariogram for 2 perpendicular directions and semivariogram surfaces

Predictions with Anisotropy and Simulations

- **Anisotropic Spatial Variation – Example Elevation in a valley**
  - **Angles measured clockwised from 0 degree at the North**





Anisotropy parameters

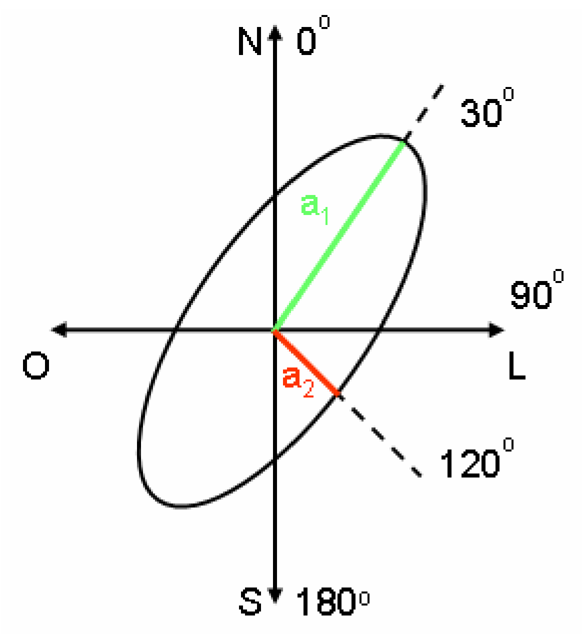
Anisotropy factor (Fa) $Fa = a_2 / a_1$

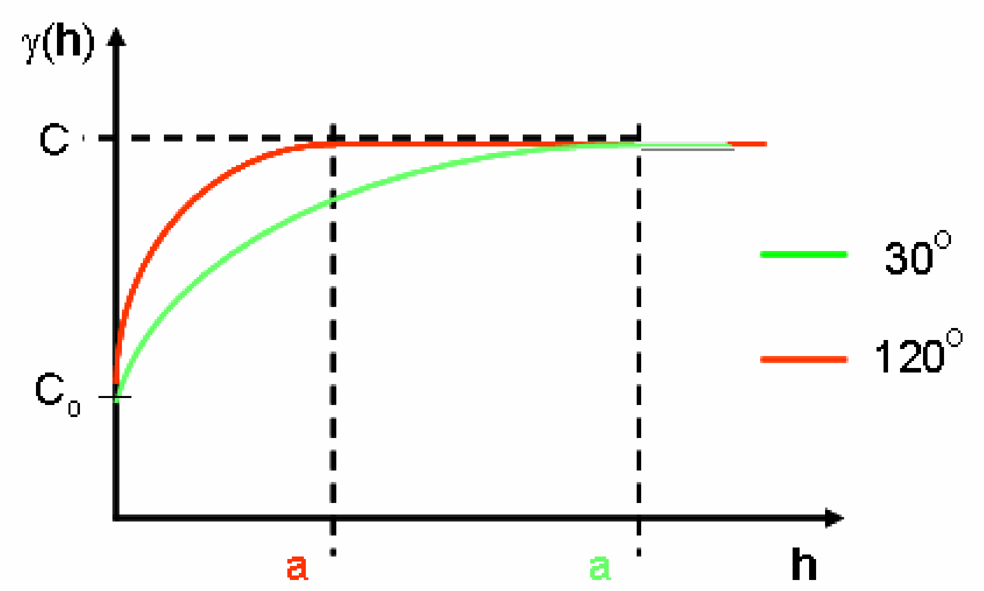
Anisotropy angle (Aa)

Angle of the greater continuity (30^0 in this example)

Predictions with Anisotropy and Simulations

- **Anisotropic Spatial Variation – Anisotropy Types**
  - **Geometric Anisotropy**
    - 2 semivariograms with same model function, same sills and different ranges



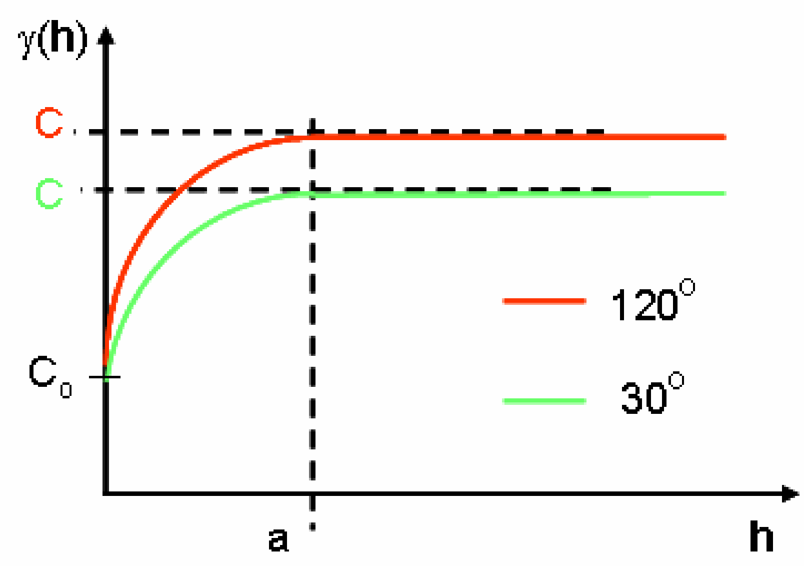


Predictions with Anisotropy and Simulations

- **Anisotropic Spatial Variation – Anisotropy Types**

- **Zonal Anisotropy**
  - 2 semivariograms with same model function, same ranges and different sills

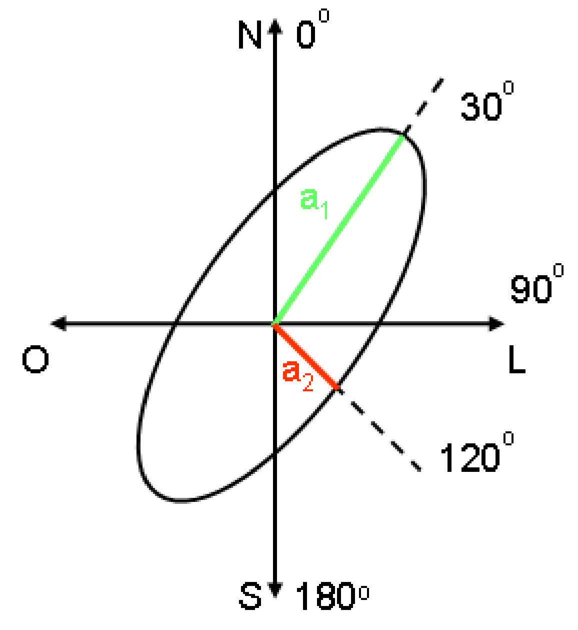
less frequently found for natural phenomena

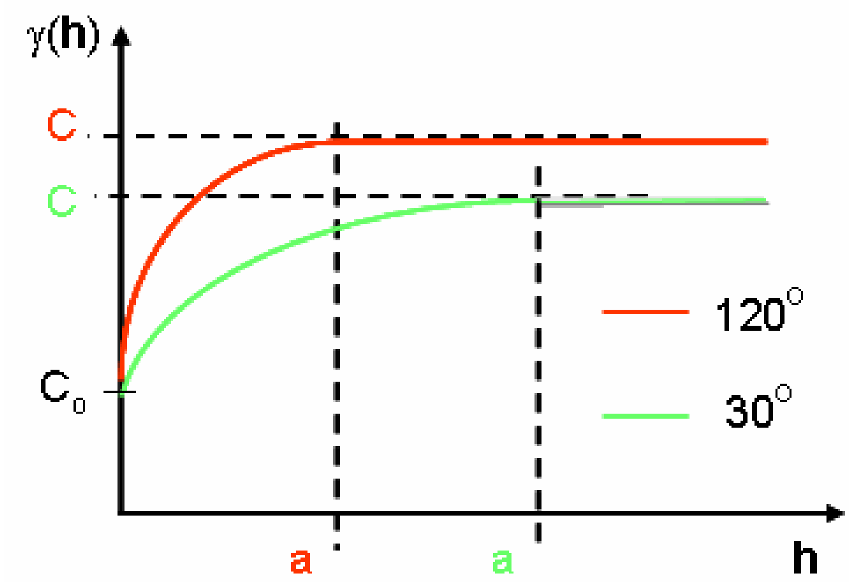


Predictions with Anisotropy and Simulations

- **Anisotropic Spatial Variation – Anisotropy Types**

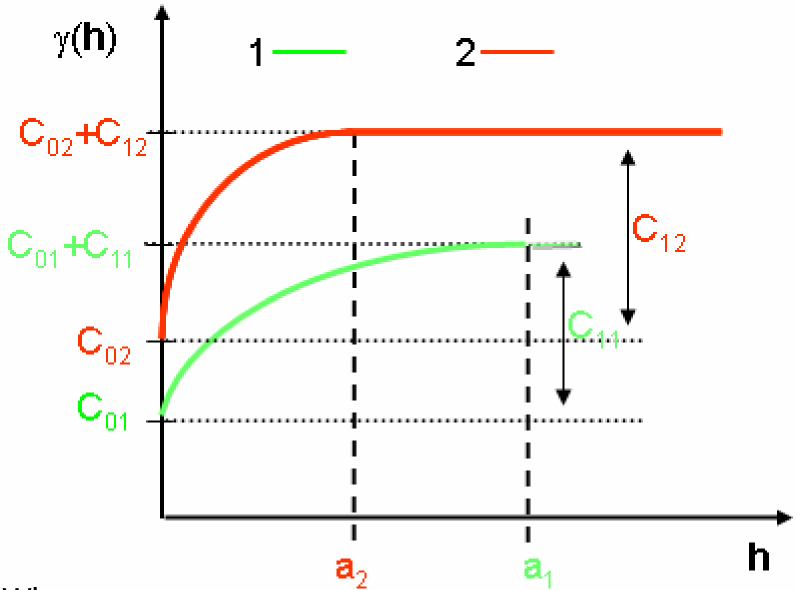
- **Combined (Geometric + Zonal) Anisotropy**
  - 2 semivariograms with same model function, different sills and ranges
  - it can also have different nugget effects, but is not common





Predictions with Anisotropy and Simulations

• **Modeling Anisotropic Semivariogram –** defining a resulting semivariogram from the two perpendicular unidirectional variograms



$$\gamma(\mathbf{h}) = C_{01} + (C_{02} - C_{01}) \cdot \mathrm{Exp}\left(\frac{\mathbf{h}_1}{a_1}, \frac{\mathbf{h}_2}{\varepsilon}\right)$$
$$+ (C_{01} + C_{11} - C_{02}) \cdot \mathrm{Exp}\left(\frac{\mathbf{h}_1}{a_1}, \frac{\mathbf{h}_2}{a_2}\right)$$
$$+ (C_{02} + C_{12} - (C_{01} + C_{11})) \cdot \mathrm{Exp}\left(\frac{\mathbf{h}_1}{\infty}, \frac{\mathbf{h}_2}{a_2}\right)$$

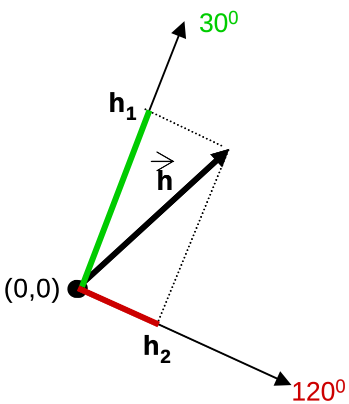
Where:

C_{01} is the nugget effect of the variogram 1 and C_{11} is the contribution of the variogram 1

C_{02} is the nugget effect of the variogram 2 and C_{12} is the contribution of the variogram 2

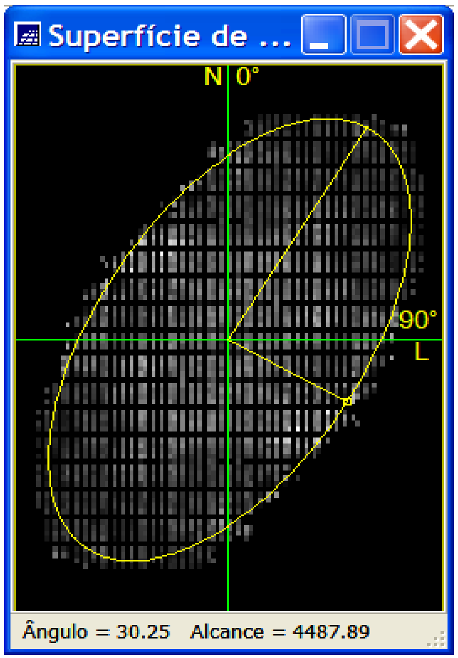
$\mathbf{h_1}$ is the module of the vector **h** in the direccion of variogram 1 (30^0 for example)

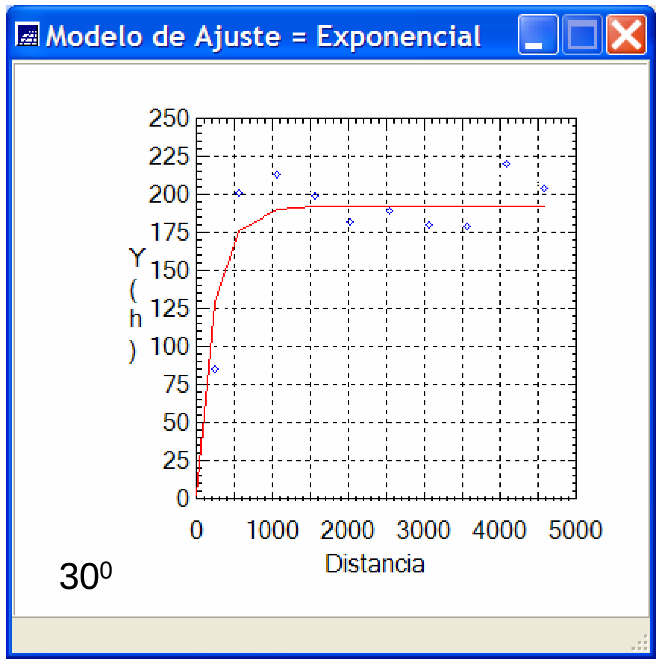
$\mathbf{h_2}$ is the module of the vector **h** in the direccion of variogram 2 (120^0 for example)

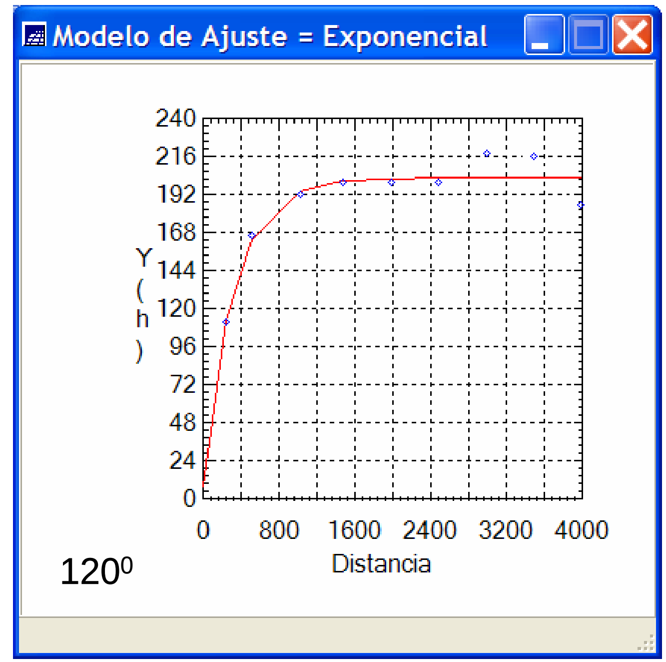


Predictions with Anisotropy and Simulations

- **Modeling Anisotropic Semivariogram –** Example in the laboratory





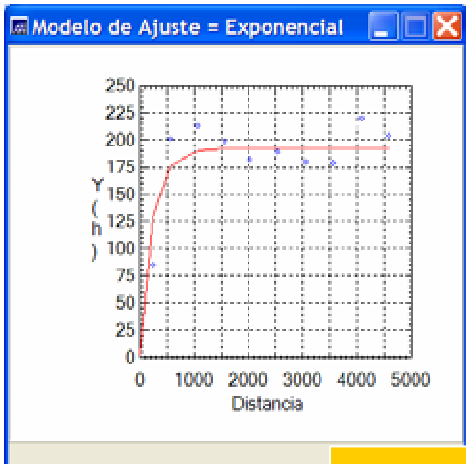


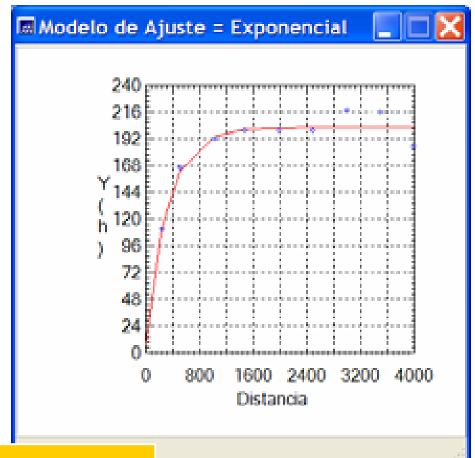
$$\gamma(\mathbf{h}_{30}) = 6.843 + 194.880 \cdot \mathrm{Exp}\left(\frac{\mathbf{h}_{30}}{961.804}\right)$$

$$\gamma(\mathbf{h}_{120}) = 1.106 + 190.084 \cdot \mathrm{Exp}\left(\frac{\mathbf{h}_{120}}{674.548}\right)$$

Predictions with Anisotropy and Simulations

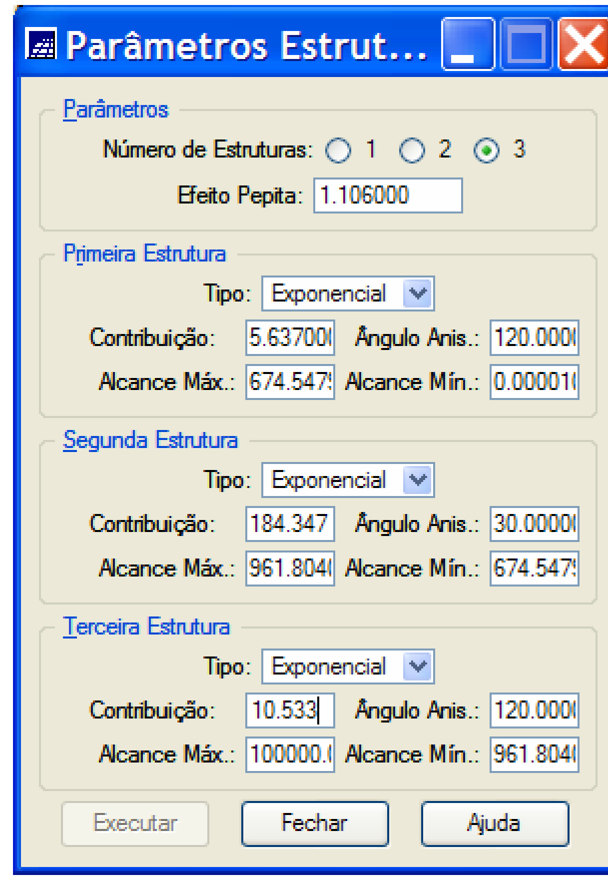
- **Modeling Anisotropic Semivariogram –** Example in the laboratory





COMBINATION

$$\gamma(\mathbf{h}) = 1.106 + 5.637 * Exp\left(\frac{h_{30}}{\varepsilon}, \frac{h_{120}}{674.548}\right) + 184.347 * Exp\left(\frac{h_{30}}{961.804}, \frac{h_{120}}{674.548}\right) + 10.533 * Exp\left(\frac{h_{30}}{961.804}, \frac{h_{120}}{\infty}\right)$$



Predictions with Anisotropy and Simulations

- **Kriging prediction**

Summary

Following Journel, 1988: $\mathbf{K}.\boldsymbol{\lambda} = \mathbf{k} \Rightarrow \boldsymbol{\lambda} = \mathbf{K}^{-1}\mathbf{k}$

$$\begin{vmatrix} \lambda_1 \\ \lambda_2 \\ : \\ \lambda_n \\ \alpha \end{vmatrix} = \begin{vmatrix} C_{11} & C_{12} \ldots\ldots & C_{1n} & 1 \\ C_{21} & C_{22} \ldots\ldots & C_{2n} & 1 \\ : & : & : & : \\ C_{n1} & C_{n2} \ldots\ldots & C_{nn} & 1 \\ 1 & 1 \ldots\ldots & 1 & 0 \end{vmatrix}^{-1} \begin{vmatrix} C_{10} \\ C_{20} \\ : \\ C_{n0} \\ 1 \end{vmatrix}$$

The elements of the matrices are evaluate by the relation: (*Journel, 1988*):

$$C_{ij} = C(\mathbf{0}) - \gamma(\mathbf{h}) = C_0 + C_1 - \gamma(\mathbf{h})$$

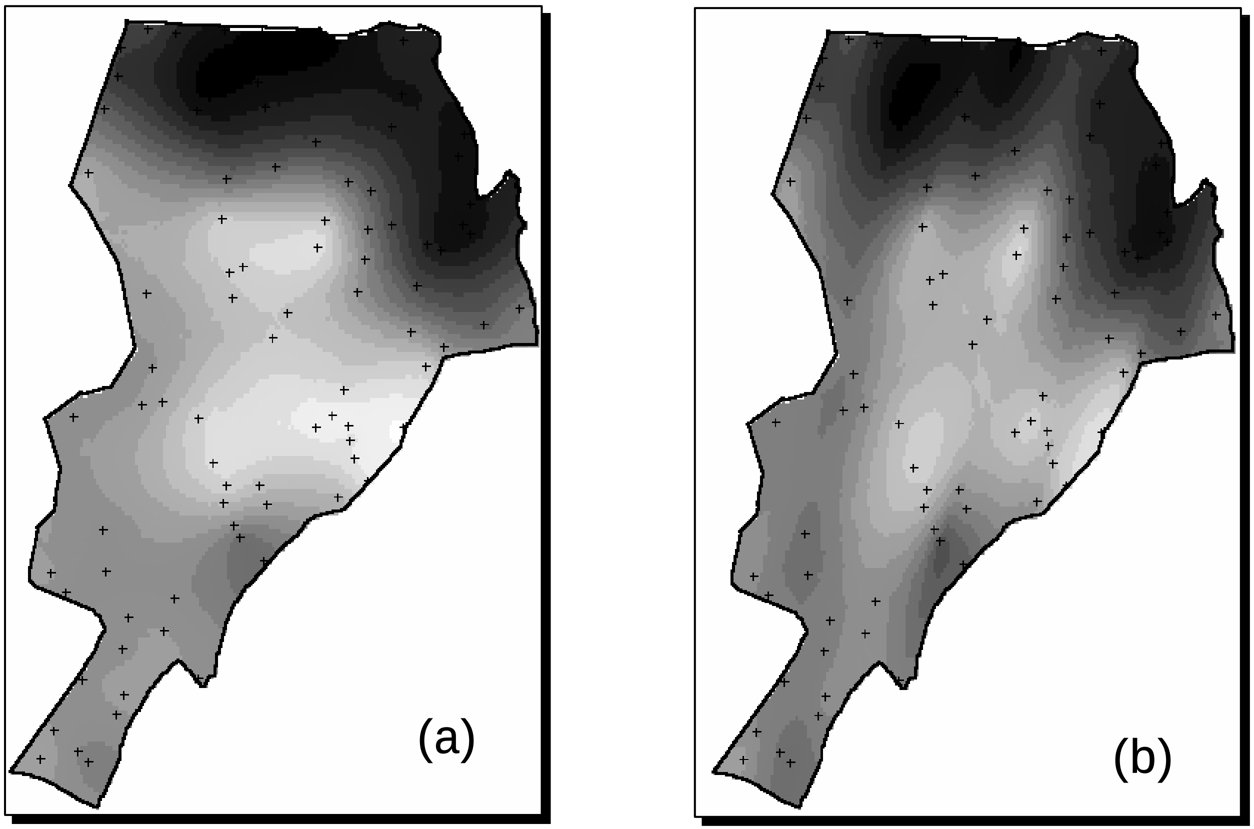
Replacing the C_{ij} values in the matrices one find the weights λ_1, λ_2, ..., e λ_n.

The Kriging Estimator is given by: (*Journel, 1988*): $Z^*_{\mathbf{x}_0} = \sum_{i=1}^{n} \lambda_i Z(\mathbf{x}_i)$

The Kriging Variance (*Journel, 1988*): $\sigma^2_{ko} = (C_0 + C_1) - \lambda^T k$

Predictions with Anisotropy and Simulations

- **Kriging prediction – isotropic x anisotropic modeling**

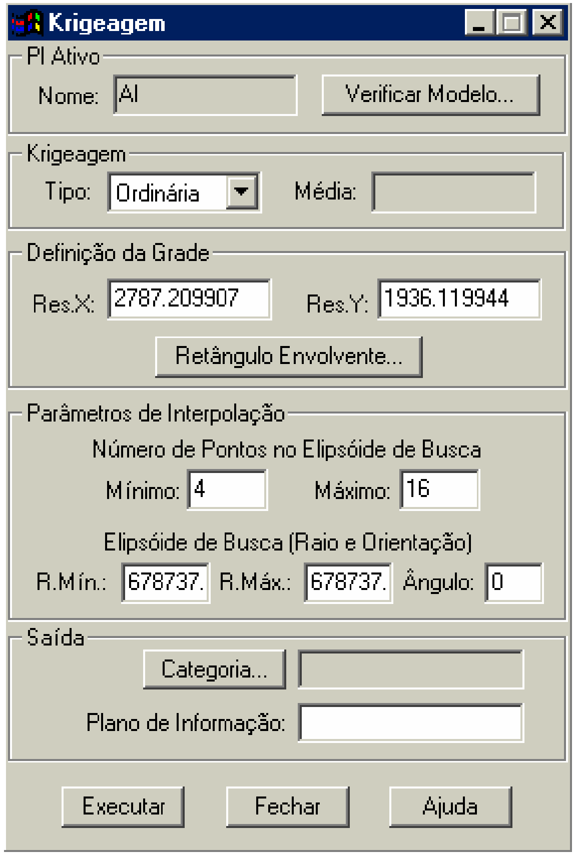


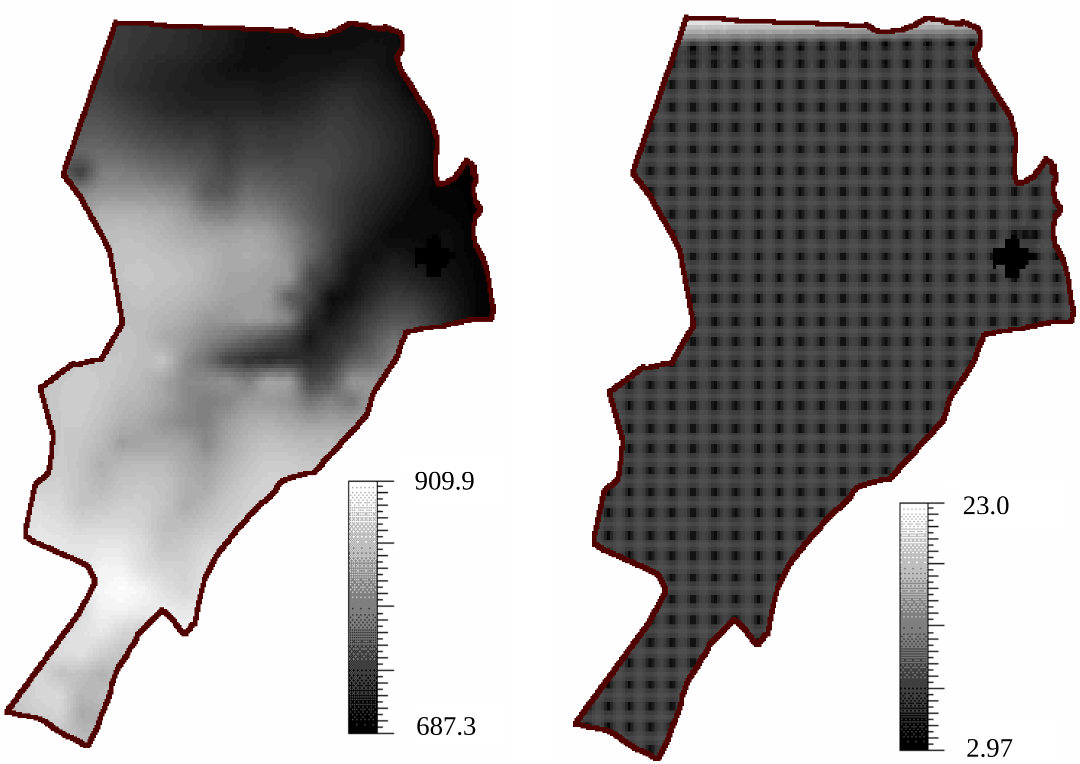
Anisotropy angles 17^0 and 107^0

Examples of evaluation of the means values by kriging considering (a) isotropic and (b) anisotropic spatial variations

Predictions with Anisotropy and Simulations

- **Kriging prediction – estimate means and variance of the estimates**



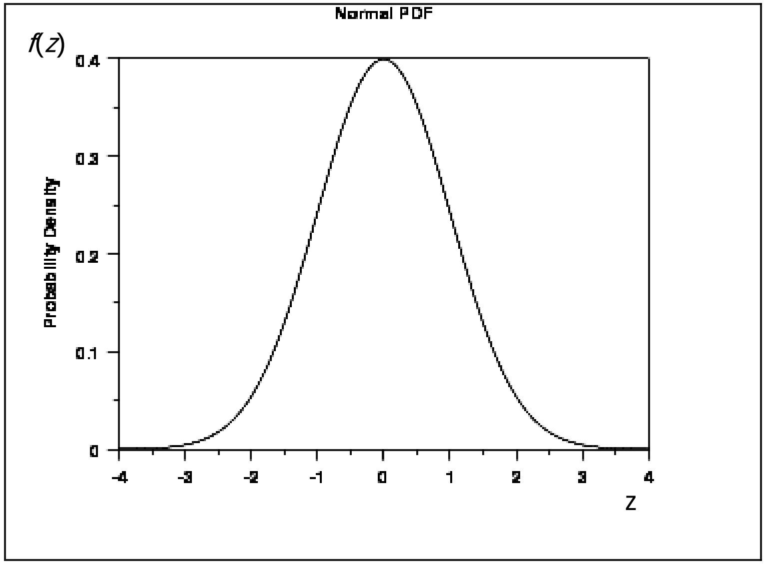


Maps of kriging means and kriging variances

Predictions with Anisotropy and Simulations

• **Simulations –** allows to get realizations from a stochastic model representing a Random Variable or a Random Field.

• **Gaussian Simulation -** Using the hypotheses that the mean and the variance (or standard deviation) evaluated by kriging are parameters of gaussian distributions one get (at each location for example) the following distribution equation (and graph):



$$f(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}[(z-\mu)/\sigma]^2}$$

If the distribution is normalized μ=0 and σ=1

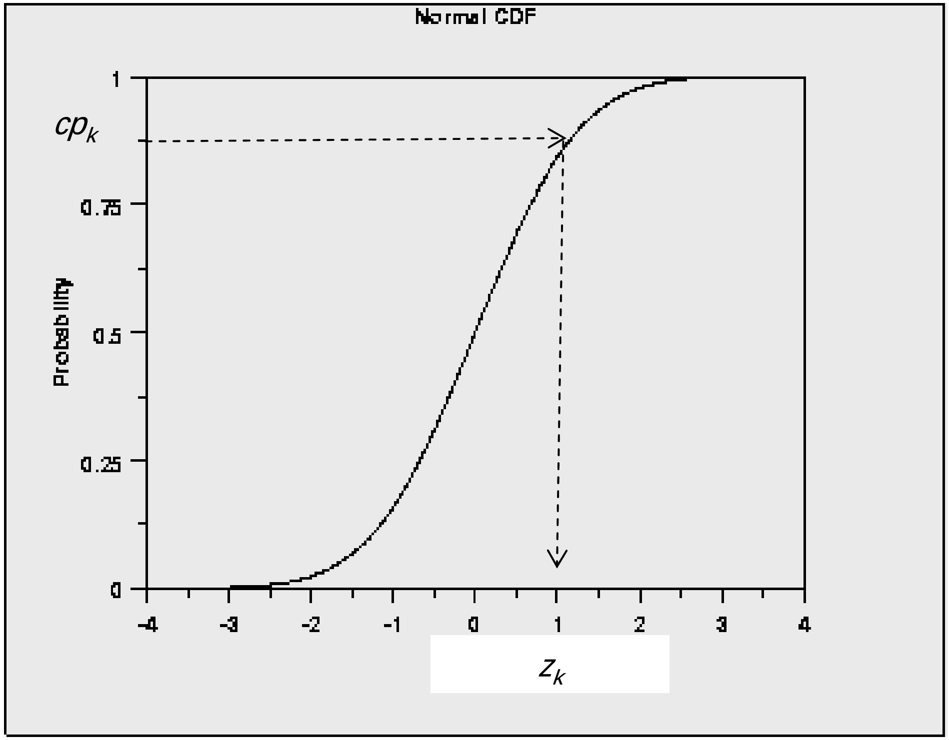
$$f(z) = \frac{e^{-z^2/2}}{\sqrt{2\pi}}$$

Predictions with Anisotropy and Simulations

- **Simulations –** the process of getting realizations of the Gaussian distribution Uses the cumulative distribution function (cdf) and a random number generator.

- **N** realizations of each RV *Z* are obtaining repeating *n* times the steps:
  1. Generating a random number between 0 and 1 (*cp* - cumulative probability value)
  2. Mapping the *cp* to the *z* value using the Gaussian cdf defined by the given μ_z and σ_z parameters.

- **Problem: How can I prove (or verify) the hypothesis that the distribution in each estimated location follows a Gaussian (Normal) distribution?**



Predictions with Anisotropy and Simulations

- **Problems with geostochastic procedures**

The main drawback of using geostatistic approaches is the need of work on variogram generations and fittings. This work is interactive and require from the user knowledge of the main concepts related to basics of the geostatistics in order to obtain reliable variograms.

The kriging approach is an estimator based on weighted mean evaluations and is uses the hypothesis of minimizing the error variance. Because of these the kriging estimates create smooth models that can filter some details of the original surfaces.

Predictions with Anisotropy and Simulations

- **Advantages on using geostochastic procedures**

  - Spatial continuity is modeled by the variogram
  - Range define automatically the region of influence and number of neighbors
  - Cluster problems are avoided
  - **It can work with isotropic and anisotropic phenomena**
  - Allows prediction of the Kriging variance
  - **Allows simulating (get realizations from) random variables with normal distributions.**

Summary and Conclusions

Summary and Conclusions

- Geostatistic estimators can be used to model spatial data.
- Geostatistics estimators make use of variograms that model the variation (or continuity) of the attribute in space.
- Geostatistics advantages are more highlighted when the sample set is not dense
- Current GISs allow users work with these tools mainly in Spatial Analysis Modules.

Predictions with Anisotropy and Simulations

Exercises

- Run the Lab4 that is available in the geostatistics course area of ISEGI online.
- Find out if the variation of your attribute is isotropic or anisotropic. Model the anisotropy if it exists.

Predictions with Deterministic Procedures

END

of Presentation